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MECHANICS

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A MANUAL
OF
MECHANICS

AN ELEMENTARY TEXT-BOOK

DESIGNED FOR STUDENTS OF APPLIED MECHANICS

BY

T.homas M.inchin GOODEVE, M.A.

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MANUAL OF MECHANICS.

CHAPTER I.

INTRODUCTORY.

It is the province of geometers to discuss the nature of lines, surfaces, and solids, and to present the idea of space. When that has been done, we may further say that matter is anything which can fill a portion of space and is capable of influencing our senses.

Definition: Force is any cause which moves, or tends to move, any portion of matter.

In order to conceive the existence of a force we must also conceive that there is something upon which it can act, and accordingly we regard matter, not only as something which can be perceived by the senses, but also as anything which can be acted upon by, or can exert, force.

Our ideas of matter and force coexist together. We shall soon learn to connect the idea of force with that of matter in motion, and shall further regard matter as the depository of force.

When the tendency which force exerts to move a body is counteracted so that the body remains at rest, the force is commonly called *a pressure*.

In the science of mechanics, as in other sciences, certain facts of observation are first recorded; then arises some one who reasons upon a group of facts, and connects them together by a natural law.

When this law is once clearly seen and appreciated, the position so taken up is never lost, the law remains for the guidance and benefit of future generations, and year after year some new and useful application of it may be discovered.

The law of universal gravitation, as it is termed, is an example of such a natural law, and was discovered and laid down by Newton.

No truth in science can be deemed to rest upon a more sure foundation. The law may be stated in the most general terms, and is a rule without any exceptions. It is the following:

Every particle of matter is endowed with a power of attracting or drawing towards itself every other particle, wherever situated; and, further, the attractive power of a particle at different distances diminishes in the same proportion as the square of the number expressing the distance increases.

Such attractive power is inherent to matter, and can neither be given nor taken away, nor is there anything, which we can do, that will influence or modify it in the smallest degree.

The attractive power residing in small bodies is so feeble that we do not perceive it. Moreover, it resides in the centre of a body or particle. Two lumps of sugar placed in contact on a table do not adhere or pull each other together. Also, the attraction emanates from their centres, which may be an inch apart, and the power is too small to be felt under these circumstances.

But powder the sugar so as to bring the centres of attraction very close together, and you can build up a wall with the dust by paper knives. The same thing is apparent with lead. Take two leaden bullets, cut a slice off each with a sharp penknife and press the clean-cut surfaces strongly together, when the bullets will adhere from the holding action of the parts in contact.

Every one is aware that matter exists in three forms, viz. as solids, liquids, and gases.

It is extremely difficult to define a solid; nothing is gained by merely giving examples. Thus we say that a leaden ball is a solid, that a steel ball is a solid, that a ball of vulcanised india-rubber is a solid, yet each of them is quite unlike the others, and the question is, what is there common to all three which distinguishes them from some well-known liquid, such as water.

In order to answer this question, put each ball in succession between the parallel jaws of a vice, and compress it. The lead will yield a little, but will afterwards support the pressure. The steel will scarcely yield at all. The india-rubber ball will lose its shape and bulge out at the sides, but will soon adapt itself to the pressure, and will support it without any external aid.

Hence we distinguish solid bodies as those which can resist a change of shape in some degree, according to the following definition.

Definition: Bodies which can support a longitudinal pressure, however small, without being supported by a lateral pressure, are called *solid bodies.*

Thus, a pillar of iron is a solid body. It will sustain a great pressure from a weight resting on its top—that is, a pressure in the direction of its length, or a longitudinal pressure—without any lateral support along its sides. It is this property which makes it useful in building, and we often see iron pillars used for supporting overhanging galleries.

As regard liquids, if we pour any liquid, such as water, into a tumbler, it will lie at the bottom and be separated by a distinct surface from the air above it. Also this surface will be a horizontal plane.

Again, a liquid may be divided in any direction, and its particles may be stirred about and moved amongst each other with the smallest conceivable effort. The ease with which the particles of coloured water will diffuse themselves through clear uncoloured water is very striking, as any one may observe on mixing a solution of sulphate of copper with clear water. We could not mix blue and white powders in the same manner.

Our next observation is that a liquid differs essentially from a solid in being destitute of the power of sustaining pressure unless it is supported laterally in every direction. In other words, it is not possible to take up a portion of liquid and squeeze it in a vice as we could squeeze a piece of iron. The moment we put pressure on any part of the surface of water inclosed in pipes or vessels, that pressure is carried on by chains of easily moved particles to every other point of the liquid and is felt everywhere in thousands of different directions at once. We shall return to this subject hereafter; at present it may suffice to say that water is a fair type of liquids, and that its principal mechanical property is that of transmitting pressure equally in all directions.

As to the air which lies above the water in the tumbler, that is a gas, and perhaps the primary distinguishing property of a gas is that of *indefinite expansion.*

Suppose that we have a large closed glass jar which is perfectly empty, and that we can introduce within it a small quantity of the air around us, so much (for example) as would fill a thimble. This quantity of air would, when so introduced, at once expand or enlarge itself so as to fill the whole jar, and would press against its interior surfaces in all directions, from below upwards, as well as from above downwards. It is such a power of expansion accompanied by pressure which distinguishes a gas.

Liquids and gases are both included under the general term *fluids.*

ART. 1. The question now arises: In what way are solids, liquids, and gases built up or formed? At this stage the student can scarcely hope to gain a clearer conception of the constitution of matter than that which presented itself to the mind of Newton, who wrote :

'It seems probable to me that God, in the beginning, formed matter in solid, hard, impenetrable movable particles, of such sizes and figures, and with such other properties, as most conduced to the end for which He formed them, and that these particles, being solids, are incomparably harder than any porous bodies compounded of them—even so very hard as never to wear or break in pieces.' (See Daniell's 'Chemistry,' p. 7.)

It is here assumed that the particles or atoms which form solids, liquids, and gases are all equally hard, solid, and impenetrable, and that the different qualities or properties which we observe in different substances are due to the manner in which these atoms are held together. The individual particles which make up the point of a diamond, the down of a feather, the web of a spider, the soft balmy air of summer, are all so incomparably hard and impenetrable that no human power can wear away or break up or divide them.

2. Since we cannot isolate the separate atoms of any substance, it is impossible to prove directly the statement of Newton. The extent to which matter can be subdivided is a subject which has always excited interest, and Dr. Miller, the chemist, has given the following example of the subdivision of gold. It appears that a grain of gold may be hammered out so as to cover an area of 49 square inches. One square inch of the leaf may be easily attached to a plate of glass by floating it on water upon the glass and then allowing the liquid to drain off. Very fine lines can then be ruled on the gold leaf, as many as 10,000 in an inch, and if two sets of lines be ruled at right angles to each other, we divide the square inch into 100,000,000 pieces, each of which can be recognised in a microscope. Hence the single grain of gold has been subdivided into 4,900,000,000,

fragments. Yet we are quite sure that we have not approached the possible limits of subdivision, for we can obtain a much thinner film of gold leaf by coating silver wire with gold and then drawing out the gilt wire.

3. Instances of this kind could be multiplied, but we have only space for one more illustration.

Place a tiny drop of blood on the point of a needle and examine it by a microscope, when a number of minute spherical globules will be seen, each of which is less than $\frac{1}{3000}$ of an inch in diameter. These globules are not atoms, for they can be broken up by chemical action. But we can go further, for it is found that there are distinct animalculæ no larger than these tiny globules. These minute creatures give evidence of circulation, of nutrition, of the same character of life which is observed in the larger animals; they are essentially beings, and beings made up of parts.

Our conclusion, then, is that we must assume the existence of actual atoms, which are incomparably smaller than those last minute particles which we can seize upon and distinguish by the aid of the most powerful microscopes.

4. The conception of Newton has been modified in recent times, and the atoms to which he referred have been regarded as grouped together in definite collections called *molecules.* Thus it is not believed that a particle of water is a single atom, being one and indivisible; but, on the contrary, it is thought to be a compound of at least two atoms, each of which is taken from a different substance, and is in itself indivisible. So long as these separate atoms are grouped together into one we may have the substance water, when they are separated we have other substances known as hydrogen and oxygen gases. A molecule of water is a definite thing—it replaces an atom in Newton's statement. But it is not an atom, for it may be broken up by certain natural operations. According to Newton's view, the ultimate atoms of matter are *indestructible*, they are capable of combining amongst each other in a multitude of different

ways, producing substances of the most diverse properties, but no one of them is ever destroyed.

5. In searching for the atoms of a solid, we may find them in very unexpected places. Thus, dissolve some white sugar in water so as to form a thick transparent syrup. This appears to the eye clear and colourless, but it is really full of black atoms of carbon in prodigious numbers. For if we pour some of the solution into a beaker standing on a plate, and add sulphuric acid and stir up the mixture, it will froth up and boil over on to the plate in a mass of black carbon.

Again, dissolve some nitrate of silver in distilled water, we shall obtain a clear liquid as bright and sparkling as the purest water, but which is really full of silver atoms. If we now pour a little aldehyde (a substance formed from alcohol) into a test tube, add some of the solution of silver with a little ammonia, and heat the tube over the flame of a spirit lamp, the metal will be deposited in a bright metallic state on the inside of the tube, forming a silver reflector.

6. Again, let us fix our attention on ice, water, and steam. As is well known, a mass of ice may be thawed into water, and the water may be boiled and converted into steam, which when not allowed to escape into the air remains a transparent invisible gas. The same identical molecules are present in each case, and we have therefore one and the same mass of matter in each of the three forms—solid, liquid, and gaseous.

Other gases, such as oxygen, or the air we breathe, have been of late converted into liquids though not into solids, but there are some gases, such as carbonic acid, which may be readily liquefied and afterwards *frozen* into a solid.

7. **Porosity.**—All substances are *porous.* Pores are the intervals of empty space which exist in the interior of bodies; thus the holes in a sponge are pores of great size. But in addition to the larger holes which are apparent to the eye, there are in a piece of sponge innumerable smaller channels and passages which can only be seen by a magnifying glass.

If we compress the sponge, it becomes smaller, but no compressing machine, however powerful, would enable us to get rid of the porous spaces altogether.

All cavities or empty spaces in a body, which can be perceived by a microscope, we describe as *sensible* pores; but it is believed that substances are porous in a different sense, and that their different molecules are separated by intervals or spaces which no optical instrument will enable us to detect.

Porosity of the kind to which we now refer is distinguished as *physical porosity.*

It appears that every substance in nature is compressible more or less; that is to say, it will become smaller in bulk when subjected to pressure all round its surface. Innumerable experiments have been made, and the result is that every substance in nature will yield to a compressing force.

The reasoning is very briefly this:

Assume the existence of atoms which in themselves are hard and incompressible; if the substance made up of atoms be compressible it must be porous, that is to say there must be empty spaces of some kind between the respective atoms.

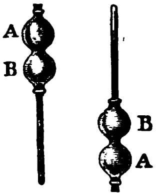
If this were not so, the body would be like a box full of hard balls; when subjected to pressure there would be nothing to give way, for the separate atoms are incompressible, and there would be a complete line of atoms in contact in the direction of every pressure.

8. The porosity of solids will be readily admitted: a piece of wood sucks in water and becomes wetted throughout, as we see by cutting it in halves. Also a lump of sugar will suck up water and become wetted and dissolve, but a lump of gold will not. Yet the gold too is porous. Many years ago a learned body of men in Florence inclosed water in a gold globe and compressed the globe out of shape, when the gold was not broken but water was forced through in a fine dew. The liquid had passed through pores in the metal.

But a substance may be porous when it is impossible to force water through it. Glass is such a substance. No pressure has yet been sufficient to force water through glass, yet glass is compressible, and therefore porous, as we argue.

One test of compressibility, and often the easiest one to put in practice, is to try the power of the substance to produce a musical note. Thus, take a glass rod or tube some three or four feet long, grasp it firmly in the centre between the finger and thumb of one hand, and rub a piece of damp flannel, with moderate pressure, in slow even strokes to and fro along the glass. It will be easy to put the rod into vibration, and a loud musical note will be heard. This musical note is due to the fact that each half of the rod is alternately compressed and elongated, as may be rendered apparent by bringing a light pith or wooden ball, suspended by a thread, in contact with either end of the rod. The ball will dance violently to and fro under the successive blows given by the vibrating end.

9. Again, water is porous, it will absorb air or gases, as we see in a bottle of soda-water. There is a simple experiment on this subject. Take a vessel of glass with two globes, A and B, and a stem, as shown. Fill the vessel with water up to the level of the neck between A and B, and then fill the globe A quite full of alcohol and put in a cork. With care the liquids will not mix. Now invert the vessel, when the alcohol will rise through the water. A number of bubbles will be seen, although there is no chemical action. The two liquids when mixed occupy a smaller space than before mixture, and the porosity is shown by the empty space in the tube when the bubbling has ceased.



The final and conclusive test is derived from compression. Is the liquid compressible? If so, it must be porous.

As a general rule, liquids are much less compressible than solids; but they all yield to pressure, and can be made also to conduct sound, which they can only do in virtue of being alternately compressed and expanded.

It is estimated that a pressure of 15 lbs. on the square inch would compress

Water . . .	about	50	millionths of its	volume at	32° F.
Mercury . .	„	3	„	„	„
Ether . . .	„	111	„	„	„

10. **Elasticity** is that property whereby the particles of a body return to their original positions after they have been displaced.

A billiard ball is *elastic.* Smear a thin coating of lamp-black and grease on a level slab of marble. Touch the slab with the ball, when a very small round black spot will appear on its surface. Now throw the ball down strongly on the slab, when it will rebound, and, although it has recovered its shape perfectly, there will appear on it a circular black spot nearly as large as a threepenny-piece. The ball has been sensibly flattened at the point of impact, but has immediately recovered its spherical form. We say, then, that a *perfectly elastic* body recovers its original shape when any cause which has changed its form or volume ceases to act.

In this sense all gases are perfectly elastic, and so are all liquids. After compression they do not remain of diminished bulk, but recover at once their former volume when the pressure is relieved. There is no limitation to this statement: whether the compression be great or small, the recovery of volume is perfect.

Solids, on the contrary, are imperfectly elastic, and in some cases exhibit scarcely any appearance of elasticity. As a rule, however, it may be said that every solid will bear some minute derangement of form and yet recover its shape. So far it is perfectly elastic; but sooner or later a permanent change will take place, and the limit of alteration of form with subsequent recovery will be exceeded.

Thus, iron is perfectly elastic, and wood very nearly so, within the limits to which we strain their powers in any structure. Engineers never subject iron to a pressure calculated to produce a permanent alteration of form.

In wrought iron this limit is estimated at a pull of eight to ten tons per square inch of section, the breaking pull being twenty-six to thirty-two tons. When iron assumes the form of steel the limits of elasticity are much extended.

11. The mechanical tests of perfect elasticity are:

(1) Recovery of form.

(2) Each additional unit of stretching weight produces the same increase of length. In Barlow on the 'Strength of Materials,' the following example is given:—

A bar of wrought iron 1 square inch in section was suspended at one end, and a weight of 4 tons was hung upon it, the length being carefully observed. Then weights of 5, 6, 7, 8, 9 tons were suspended, and the extension for each additional ton weight was found to be 120, 110, 120, 120, 120 millionths of the length of the bar, the rule being followed with only one small deviation in the case of the 6 ton weight.

A like test would be true for compression; hence we infer that:

Resistance to extension varies as the extension;
" " compression " " compression.

12. When a body can be set into vibration so as to give out a musical note, we have the best proof of the elasticity of the material of which it is composed. The recovery of form after extension or compression must be perfect or vibration cannot go on.

Steel and glass are the only substances which become hard and elastic by sudden cooling at a red heat. This process renders them wonderfully hard and elastic, but correspondingly brittle or liable to break. In truth, unless glass vessels were annealed, as it is termed—that is, allowed to cool slowly by passing them along an open chamber, heated at one end and cool at the other—they would be so brittle that we could not use them. In like manner brittleness may be greatly removed from hardened steel articles by tempering, that is, by raising them to a temperature ranging from 430° F. to 560° F. and then allowing them to cool very gradually. A sword must be tempered or it would break at the first blow.

The general effect of heat is to diminish the elasticity of metals, and finally to remove it altogether. This may be shown by heating a brass bell over the flame of a spirit lamp. When struck it will not ring, but the blow sounds as if it fell upon wood.

There are curious points of difference in different substances. Thus, heat and moisture together develop perfect elasticity in a feather, as may be seen by crushing a feather in the hand and then holding it in the steam issuing from the spout of a kettle of boiling water.

Also Chinese gong metal is hardened by being kept at a red heat for some time and then cooled very slowly. Unlike steel, it is made soft and malleable by being brought to a red heat and then suddenly cooled.

13. **Plasticity.**—The next property we refer to is that of *plasticity*.

Many solids are hard to the touch, and yet will yield to severe pressure, allowing their molecules to slip over each other and take up new positions. New forms or surfaces can then be impressed upon them.

It is in virtue of this property that such a thing as a Britannia metal teapot is raised into form from a flat circular sheet by burnishing or rubbing the metal into shape. Coins are stamped in like manner by hard steel dies. To get an idea of the meaning of the term *plastic*, take two clean pieces of sheet lead, and place a half-sovereign between them, then strike a hard blow with a hammer, and there will be a perfect impression of the coin upon the lead.

The metal steel, as it appears in the blade of a knife, has no plasticity, and if we were to put the half sovereign on a steel blade, and then to strike a hard blow with a hammer, no impression would be made on the steel, but the coin would be damaged. Hence a steel die may be used for stamping a flat smooth disc of gold and converting it into a standard coin.

14. **Tenacity.**—When the atoms of a body are held together so strongly and firmly that no ordinary amount of force will sensibly increase or diminish the distance between them, the body is said to be *rigid*. There is no such thing as a perfectly rigid body in nature, but we often suppose a body to be rigid when examining its behaviour under the action of forces.

The power with which the atoms of some substances hold together is remarkable. Thus, Mr. Brunel, who designed the suspension bridge across the Avon at Clifton, stretched a steel wire, 700 feet long and about 1½ square inch in section, between the two piers of the bridge, and many persons crossed in a chair hanging on this slight support.

In fact, a wire of steel, 1 square inch in section, will support a load of 60 tons hung at one end, whereas a similar wire of forged copper would support about 15 tons. If the wire were of tin it would support only 2 tons, and the plastic metal lead would sustain no more than four-fifths of a ton under like conditions.

It is interesting to note the action of heat upon metals in softening them and rendering them plastic. Take a copper wire about the thickness of a stout sewing thread; it will support a considerable weight (say 28 lbs.). Bring the flame of a spirit lamp against the wire, and the weight drops at once. The atoms of copper can no longer resist the pull to which they are subjected.

The student will naturally ask for an explanation, and at the present day we should account for it in a different manner from those who wrote fifty years ago, and should state our belief that the particles of copper are set into such violent trembling motion by the heat of the flame that their power of holding together has been almost entirely taken away.

15. **Definitions.**—It follows from the law of universal gravitation that the attractive force of the whole globe of the earth is felt as a pull upon everything on its surface.

The tendency of a body to fall is familiar to every one, and arises in this way : Every particle composing the earth attracts the body, and the effect resulting from all these attractions is that the body is drawn towards the centre of the earth or very nearly so.

That such attraction really exists is evident from the consideration that the earth is round and that bodies always fall in directions perpendicular to its surface.

The terms '*gravity*' or '*the force of gravity*' are used to express the attractive force of the whole earth upon bodies on its surface.

Hang up in a frame a small knob of lead by a fine string ; this is an apparatus used by carpenters, and the string forms a so-called *plumb-line*, that is, a line pointing in the direction in which the force of gravity pulls the lead. This is, strictly, a line drawn perpendicular to the earth's level surface, or what would be its surface if there were no mountains and valleys.

The direction of the plumb-line is *vertical*, and a *horizontal plane* where the observer stands would be any plane perpendicular to this vertical line. All lines at right angles to the vertical line are called *horizontal* lines.

Now cut the string, and the piece of lead will fall in the exact line of direction of the string, that is, in a vertical line.

16. *Definition* : The *weight* of a body is the amount of force which acts upon it on account of the earth's attraction of gravity.

Weight is not an inherent property of matter ; the same quantity of matter may slightly differ in weight even at different points on the earth's surface.

In all measurements we begin by taking certain standards of reference which we call *units*; thus in measuring lengths we take a standard yard as the unit, the exact length of a yard having been handed down from early records. A yard is subdivided into 3 feet, or 36 inches, and in estimating lengths we find how many times the standard yard, foot, or inch is contained in the given length.

In comparing or measuring weights, the *unit of weight* is a certain piece of platinum which is deposited in the Office of the Exchequer, and is known as a *standard pound weight*, or a pound *avoirdupois.*

The pound avoirdupois is subdivided into 16 ounces or 7,000 grains.

The standard measure of capacity is a gallon, which contains 10 lbs. avoirdupois of distilled water at 62° F., the barometer standing at 30 inches.

The measure of capacity is related to the measure of length by the declaration that a gallon shall contain 277·276 cubic inches.

In measuring angles the unit is $\frac{1}{90}$ part of a right angle, called a degree, and the degree is subdivided into 60 minutes, and 3,600 seconds.

17. The motion of rotation of the earth about its axis gives us a measure of time, and a *solar day* is the interval between two successive transits of the sun's centre over the meridian, such as may be given by a sun-dial.

By reason of the variable motion of the earth in an orbit round the sun, which causes the apparent motion of the sun to vary day by day, the length of a solar day is not always the same, and cannot be taken as a unit for measuring time. Hence astronomers obtain by calculation a *mean solar day*, which is the mean of solar days, and is that to which clocks and watches are usually adapted.

The mean solar day is divided into 24 hours, each hour is subdivided into 60 minutes, and each minute into 60 seconds.

In mechanical examples the *unit of time* is always *one second*, unless some other unit is stated.

18. *Definition* : The *mass* of a body is the quantity of matter in it.

There are 2,240 lbs. in a ton, and it has been estimated that in round numbers the mass of the earth is between *five* and *six thousand millions of millions of millions* of times the mass of matter in a single ton.

The pound weight gives a convenient measure of force, which has been adopted in this country for many centuries.

19. *Definition* : Forces are measured by the weights they will support.

The *unit* is commonly a pound weight, whence a force which will support 3 lbs. is represented by the number 3.

On the same supposition—for the unit of force may be differently chosen—the *unit of mass* is the quantity of matter in a body weighing 32·2 pounds. This is an arbitrary assumption, but there are reasons for it which will be stated hereafter.

Let M be the mass of a body weighing W lbs. and let the number 32·2 be represented by the letter g.

Since a body of mass 1 weighs g lbs.

It follows that a body of mass M weighs Mg lbs.

$$\text{or } W = Mg, \text{ and } M = \frac{W}{g}.$$

Since the weight of a body arises from the force of gravity, it is the practice to distinguish the above mode as the *gravitation measure of force.*

20. **The centre of gravity.**—Hitherto we have not distinguished that point of a body at which the force of gravity acts. A body is made up of individual particles, and the weight of a body is the total force resulting from the action of the whole earth upon each of the particles, or,

in other words, from the combined action of the weight of the several particles of the body.

Later on we shall show that this total force is the sum of all such weights, and that it acts at an *invariable* point, that is, a point which does not change its position in the body when the body itself is turned round and inverted or otherwise moved.

This point is called the *centre of gravity* of the body.

The centre of gravity may be found by trial, or by theory, but its position is often inferred from the nature of the body. The centre of gravity of a body *of regular form*, such as a sphere, a cube, or the like, would always be the centre of figure ; and it has this distinguishing property, that *every* line drawn through it to meet the surface of the body both ways is *bisected* by it.

21. It is important to bear in mind the properties of the centre of gravity ; no other point in the body possesses the same properties.

(1) The centre of gravity does not change its position within the body when the position of the body is altered.

(2) The effect of the force of gravity on the body is to pull the centre of gravity towards the centre of the earth, or very nearly so.

(3) The force of gravity has no tendency to turn the body about its centre of gravity.

(4) The centre of gravity will always assume the lowest position which it can possibly take.

(5) If we suspend a body at any point the body will hang with its centre of gravity *vertically below* the point of suspension—that is, both the point of suspension and the centre of gravity will be in the same *vertical line*, the latter below the former. This follows from Rule 4, for the centre of gravity has its lowest position when it comes vertically under the point of suspension.

22. **Measure of velocity.**—In ordinary language the term *velocity* expresses the degree of swiftness or rapidity

with which a body is moving. Also, *motion* is change of position.

Conceive that a wheel is set spinning on its axis, and is then hurled forward obliquely into the air. The motion will be of two kinds, for there will be a spinning motion, which is one thing, and an onward motion, which is another thing. On account of the *spinning* or *rotatory motion* every line in the body perpendicular to the axis has the motion of a spoke in a carriage wheel, and describes a certain angle in each interval of time. It moves with a so-called *angular velocity.*

As regards the *onward motion*, it is capable of proof that the body moves as if its whole mass were collected or condensed in the centre of gravity.

Such a thing is physically impossible, but the supposition allows full scope for the employment of geometrical reasoning, and we shall therefore adopt it, and treat every moving body as if it were a dense particle, and shall not concern ourselves with its actual form or shape.

23. When a point changes its position continuously in relation to other fixed points, it is said to have a certain *linear velocity.*

In measuring linear velocity it is usual to adopt a *second* and a *foot* as the units of time and length. Also the number of feet which a point passes over in a given time is called the *space* described in that time.

Definition : The rate of change of position of a point relatively to other fixed points is the *velocity* of the moving point. In other words, velocity is rate of motion.

The velocity of a point may be *uniform* or *variable.* It is uniform when the point describes equal spaces in equal times ; it is variable when the spaces so described are unequal, or, more accurately, when the rate of change of position is continually altering.

24. In uniform motion the rate of change of position is determined by observing the number of feet described by

the moving point in one second. This is the method ordinarily adopted. If we say that a point has a uniform velocity of 20 feet per second, we mean that it describes 20 feet in each successive second of time. In other words:

The *linear* velocity of a point, when uniform, is measured by the number of feet described in one second.

Thus, if v be the velocity of a body moving uniformly,
s the space described in t seconds,
the letter v represents the number of feet described in one second, therefore $2\,v$ represents the number of feet described in 2 seconds, and $t\,v$ the number described in t seconds.

$$\text{Hence } s = t\,v, \text{ or } v = \frac{s}{t}$$

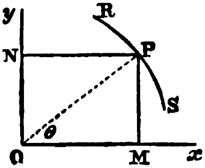
The measurement of variable velocity is beyond our scope except in one particular case which we shall discuss. It must suffice to say, that just as velocity *when uniform* is measured by the space described in a unit of time, so velocity *when variable* is measured by the space which would be described in a unit of time if the body retained throughout that unit the velocity which it has at the instant considered.

25. **Curves and coordinates.**—*Definition*: A *curve* is a line traced out by a moving point which is continually changing the direction of its motion.

When the point moves only in one given plane, the curve is said to be a *plane curve*.

Thus, a circle is a plane curve, and we shall presently discuss the motion of a point when describing a circle with uniform velocity. Circular motion lies, as it were, at the foundation of mechanism, for a steam engine or a water wheel gives the rotatory motion of a piece of shafting, and it is from a simple uniform rotatory motion of this kind that all the complicated movements observed in cotton spinning or wool combing, or the weaving of lace or figured cloth, are derived.

We begin by showing how to estimate the position of any point in a plane curve. This is done by a system of so-called coordinates. Thus let RPS be a portion of any plane curve. In the plane of RPS take any two straight lines Ox, Oy at right angles to each other, called *rectangular axes.* Draw PM and PN perpendicular to Ox and Oy respectively. It is manifest that if we know PN and PM we know the exact position of the point P. Also PN = OM, and consequently it is sufficient to know the values of OM, MP in order to determine the position of P.



Let OM $= x$, PM $= y$, then it is usual to call x and y the rectangular *coordinates* of the point P. Thus, if $x = 3$, and $y = 2$, we can at once say where P is situated, and the like when x and y have any other values.

As a matter of convenience, and because OP can be at once deduced from OM and PM by the simple rule

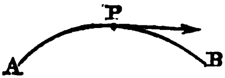
$$OP^2 = OM^2 + MP^2,$$

we have taken the axes Ox, Oy at right angles, but this is not essential, and it frequently happens that xOy is any angle arbitrarily chosen.

Again if we join OP, and know the angle POx, we can also determine the position of P.

Let OP $= r$, $POx = \theta$. Then r and θ (the Greek letter *Theta*) are called the *polar* coordinates of the point P.

26. It has been stated that a continual change in the direction of motion of a point causes it to describe a curve, and the tangent at any point of a curve is thus defined.



Definition : The tangent to a curve APB at any point P is the direction in which a point describing the curve would move at P, if it there ceased to change the direction of its motion.

27. **Graphical method.**—In this treatise a *particle*, or *material point*, or *point*, is regarded as being a portion of matter indefinitely small in all its dimensions, and yet capable of being acted on by forces, however large.

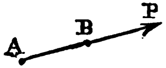
Forces may be conveniently *represented* by straight lines.

There are four things to be specified before a force can be said to be fully determined :

(1) The point upon which it acts.
(2) The line in which it acts.
(3) Its direction along that line.
(4) Its magnitude.

The point upon which a force is supposed to act must be a material point. It is not a point as defined in geometry, but may be almost regarded as such.

Conceive that a force of P units, called a force P, acts upon a point, A ; the point A will begin to move in some determinate direction as A B, and this direction is the line of action of the force P. It is usual to indicate the direction in which a force is in action by marking a small arrow-head at the end of the line. We thus determine the point upon which the force acts, the line in which it acts, and the direction along that line.



We next select some unit of length, such as an inch, $\frac{1}{4}$ of an inch, $\frac{1}{10}$ of an inch, as the case may be, and we can then represent the magnitude of the force of P units by the length of the line A B.

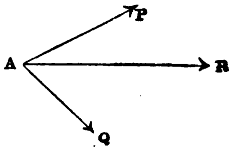
Hence the force P is fully represented by the line A B, and we remark that a force represented by A B always means a force acting at A, in direction A B, and of magnitude represented by the length A B.

We now possess a key to what is termed the *graphical* method of representing forces with a view to the solution of mechanical problems. This method enables us to show on paper any forces which are in action, each completely represented in every particular by a distinct line, and thus the mind is assisted by the eye in considering and reasoning upon the effects of these forces.

28. The graphical method of representing forces applies equally to the representation of velocity. When a point is moving with any velocity, whether uniform or variable, we may represent its velocity by a line A B (see page 21). The length of A B may represent the space actually described in one second when the velocity is uniform, and it may equally represent the space which *would be* described in one second (as previously stated) when the velocity is variable. Also the direction along A B, from A to B, represents the direction in which the point is moving at the instant considered, whether the motion be in a straight line or a curve.

Thus velocities may be represented by straight lines just as forces may be represented by straight lines.

29. **Resolution of force and velocity.**—It has been stated that force is any cause which moves or tends to move matter, and let us suppose that a particle of matter at A is acted on at the same instant by two forces P and Q. Unless these forces are equal and opposite, the point must move in some determinate direction with a definite velocity.



But a single force is competent to move a particle in a given direction with an assigned velocity. Hence there is some single force R intermediate to P and Q which, when acting on the particle, produces the same effect as the combined action of P and Q.

The single force R is called the *resultant* of P and Q, and P and Q are called the *components* of R. The process of substituting two forces P and Q for the single force R is called the *resolution* of force, and the process of finding R when P and Q are given is called the *composition* of the forces. Thus we say that R may be *resolved* into P and

Q, and that P and Q may be *compounded* into the single force R.

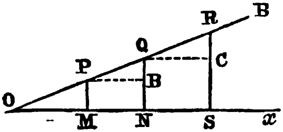
In flying a kite, for example, there are two forces acting externally, viz. the pressure of the wind and the pull of the string; a third force is the weight of the kite, which must be equal and opposite to the resultant of the two first-named forces. The kite will remain steadily supported in the air so long as this equality maintains.

If any number of forces acted at the same instant on the point A, there would still be a single force competent to move A as it actually moves. Hence the forces would have a single resultant, and the process of resolution and composition may be extended to any number of forces.

30. We have next to show what is meant by the resolution of velocity, which is a simple geometrical question.

Let a point P describe the straight line A B with a uniform velocity v.

Draw any straight line O x inclined to O B, and let Q, R, be other positions of the point P.



Draw P M, Q N, R S, perpendiculars on O x.

The point M is called the *projection* of P on O x, and it is clear that if the sunlight were shining down upon P in direction P M, so that M was the shadow of P, N would be the shadow of Q, and S would be the same of R.

Also if P B, Q C be drawn parallel to O x, we have, by similar triangles,

$$\frac{OP}{PQ} = \frac{OM}{PB},$$

But $$PB = NM, \therefore \frac{OP}{PQ} = \frac{OM}{MN},$$

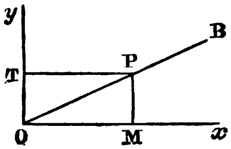
and in like manner $$\frac{PQ}{QR} = \frac{MN}{NS}.$$

Hence if P moves uniformly along O B, the projection of P, namely M, moves uniformly along O x. The motion of

M along O x is not so rapid as that of P along O B, and changes at every different inclination of O x to O B.

Thus, if the angle at O be 60°, it is easy to show that the velocity of M is exactly one-half of P.

For simplicity, let O P be the space described by P in one second. Then O M is the space described by the projection of P in one second; that is, O M represents the velocity of P resolved along O x, or is the component of v along O x.



It is evident that, if O y be drawn at right angles to O x, and P T be drawn perpendicular to O y, we may take T to be the projection of P on O y, whereby as P moves uniformly along O B, both M and T will move uniformly, one along O x, and the other along O y.

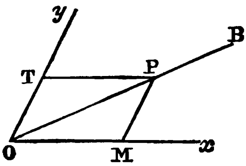
Taking O P = v, we have

O M = resolved velocity of P along O x.

O T = ,, ,, P ,, O y.

It is common to describe O M, O T as the components of v along O x, O y respectively, and to say that v is the resultant of the two velocities O M, O T.

We have drawn O x, O y at right angles to each other, but the reasoning would apply equally if O x, O y were inclined at any angle which may be selected.



Drawing P M, P T, parallel to O y, O x respectively, it is manifest that M would be the projection of P upon O x in this system of inclined axes, and T would be the projection of P upon O y.

That is, the velocity O P may be resolved into the two velocities O M, O T; and *conversely*, if there be impressed upon a point at the same time two velocities O M, O T, it will actually move with the velocity O P represented by the diagonal of the parallelogram O T P M.

31. This is the principle of the parallelogram of velocities, which may be thus stated.

Proposition : If, at the same instant, a point O receives two velocities which would be represented by the adjacent sides O T, O M of the parallelogram O T P M, the actual velocity of the point will be represented by the diagonal O P.

32. **The laws of motion and the measure of force.**—The *Laws of Motion* are certain statements respecting the effect of force in producing motion, upon which all reasoning in mechanics is or may be founded. There are three laws of motion as laid down by Newton, and the proof of these laws is of that kind which is called *inductive.* The meaning of the term *induction* is this: from certain observed facts we conjecture that a law is true, and then set about to investigate the consequences which must follow in certain particular instances if the law be true. Having done this, we observe whether such and such consequences are really matters of fact. If we find that the law can be tested in a great variety of particular cases where the observed facts would be different if the law were otherwise, we are justified in accepting the law as true, and in founding our conclusions upon it.

A few years ago it was the practice to postpone the consideration of these laws until problems relating to simple machines had been solved. But that method of teaching is now given up, and we proceed on the plan adopted in modern text-books.

33. *First Law* :—Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it may be compelled by impressed forces to change that state.

Absolute rest is not to be found in nature; the word *rest* must therefore be understood in a relative sense.

Again, an *impressed* force is merely another phrase for a force ; all *external* forces which act upon bodies are impressed forces.

This law is intended to assert the *inertia* of matter, or that quality inherent to matter whereby it has no power in itself to change its own state of rest or motion.

The conclusion that matter does not move spontaneously, or unless acted upon from without, is irresistible, and is in accordance with the experience of daily life; but the statement that any moving body, at every instant, is exerting a never-ceasing tendency to persist in one simple movement in a straight line, without any falling off or dying away of the rate at which it is moving, is not grasped without difficulty. Illustrations of the law will be noticed as they may be met with.

34. *Second Law* :—Change of motion is proportional to the impressed force, and takes place in the direction of the straight line in which the force is impressed.

Here the words 'change of *motion*' signify 'change of *quantity of motion.*'

For simplicity, we will assume the body to be *at rest* when the force begins to act; the change of motion will then be the whole motion impressed upon the body.

Since we are dealing with a body in motion, it is clear that we may consider—

1. The weight of the body to be constant, and its velocity to vary.
2. The weight of the body to vary, and its velocity to remain constant.

Take the first supposition, and let the force acting be doubled or trebled. The law says that the quantity of motion will also be doubled or trebled. But the weight is invariable, therefore the velocity must be doubled or trebled, or the quantity of motion varies as the velocity when the weight moved is constant.

Take the second supposition, and let a force acting on a pound weight for a given time impress on it a certain velocity. A second equal force acting on another pound will impress the same velocity. We shall then have a double force producing the original velocity in two pound weights.

But the law again says that the quantity of motion is doubled, and we infer that when the velocity is constant the quantity of motion varies as the weight moved. But there is a proposition known as the double rule of three, which is the following :—

Let x vary as y when z is constant or invariable ;
And let x „ z „ y „ „
Then, if both y and z vary, x will vary as yz ; or, as stated in books on algebra,

$$x = myz$$

where m is some number, whole or fractional.

Applying this proposition, we have
Quantity of motion varies
As velocity when weight moved is constant ;
„ weight moved when velocity „
It follows that when the weight and velocity both vary,
Quantity of motion = m (weight) (velocity).

Since the mass of a body is proportional to its weight, assume that

Mass of body = m (weight of body).
Hence, quantity of motion = (mass) (velocity).

The product of the numbers expressing the mass of a body and its velocity is called the *momentum* of the body, and the word momentum expresses the quantity of motion in a moving body.

Let m represent the mass of a body,
v its velocity at any instant ;
Then momentum = mv.

35. *Third Law* :—To every action there is always an equal and contrary reaction, or the mutual actions of any two bodies are always equal and oppositely directed.

Or, as more briefly stated by Newton, action and reaction are equal and opposite.

A force called into action by another force is termed a *reaction*. Thus if a man presses against an obstacle, the obstacle will, so to speak, resist that pressure by returning another pressure called *reaction*, and the law says that the return pressure or reaction is equal and opposite to the original pressure or action.

When a string is fastened to a fixed point, and we pull the string, so that it becomes somewhat rigid, it is said to be under *tension.*

But action and reaction are equal and opposite. That is to say, the pull on the string, called *action*, is equal to the opposite pull exerted at the point to which the string is fastened, or to the *reaction.*

It follows that in order to stretch a string it is necessary that two equal forces should act in opposite directions along the string.

Again, the words action and reaction apply when force sets a body in motion. The explosive force of powder in a gun is *action*, and the momentum generated in the projectile is *reaction* ; or, if we please, the momentum of the projectile being *action*, the recoil of the gun is *reaction*, and the two are equal.

36. We are now in a position to obtain a new measure of force. It has been shown that when the mass moved is constant, the force acting is proportional to the velocity impressed during its action.

But matter has the quality of inertia, whereby it opposes a definite resistance to the action of force.

Take a pound weight in the hand, and the effort made by the hand in supporting the weight is apparent. If we could suppose the pound weight removed from the earth, and existing in free space where no force was acting, it would be what we term a *free body.* If, under these circumstances, a force equal to the effort made by the hand in the former case were to act, a certain definite amount of velocity would be generated in the body in one second.

The second law, in effect, says that this amount of velocity, when we have ascertained it, will be an exact measure of the magnitude of the force which has acted.

Definition : Force is measured by the velocity generated in one second in the unit of mass in a free body.

Definition : The unit of force is a force which, acting on the unit of mass for one second, generates in it a velocity of one foot per second.

Forces measured as above are said to be expressed in *absolute measure.*

For reasons which we have not space to enter upon, the absolute unit of force is adopted only in physical or electrical treatises, and we shall not refer to it further. In this book, forces will be measured by the weights which they support. Our object in stating the laws of motion thus early, and in showing that force may be measured by its effect in overcoming the inertia of matter, or in generating velocity in a free body, is to obtain a proof of the so-called *parallelogram of forces*, a proposition by which all ordinary questions of equilibrium which come into an elementary treatise can be solved.

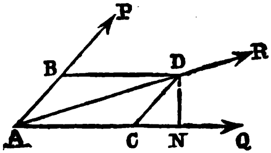
There are two fundamental principles in mechanics : (1) the principle of the lever ; (2) the principle of the parallelogram of forces. Either may be deduced from the other, and the two together give us the command over a great variety of questions. Hence the importance of the task immediately before us.

CHAPTER II.

THE PARALLELOGRAM OF FORCES.

37. Taking first the principle of the parallelogram of forces, which may be thus stated :—

Proposition : If two forces P and Q acting upon a point A have a resultant R, and if two straight lines A B, A C, represent the forces P and Q in magnitude and direction, and if the parallelogram A B C D be completed, its diagonal A D will represent the resultant R in magnitude and direction.



To prove this we refer to the laws of motion.

By the first law, it is laid down that nothing but force can change the motion of a body.

By the second law, the change of motion is in proportion to the force acting.

If then, two forces act at the same instant in different directions on a particle or body, each force will impress on it a certain velocity in the direction of its action, and, since the mass moved is constant, each velocity will be proportional to the force producing it.

Let P and Q be the forces acting, and let A B, A C represent P and Q in magnitude and direction. Then A B, A C will also represent in magnitude and direction the velocities impressed on the particle during a given time, say, one second. But A D represents in magnitude and direction the resultant of the velocities A B, A C.

Hence A D represents also the single force which produces the same effect as the combined action of the forces P and Q, or A D represents the resultant of P and Q in magnitude and direction; that is, A D completely represents the force R. Which proves the proposition.

(1). Looking generally at the diagram of this proposition we observe that as the angle P A Q diminishes, the diagonal A D becomes more and more nearly equal to $AC + CD$; or when

$$PAQ = 0, \quad R = Q + P.$$

(2). In like manner when the angle P A Q opens out to two right angles, the diagonal A D diminishes to zero, and the forces P and Q act in opposite directions in the same straight line; or when

$$PAQ = 180°, \quad R = Q - P.$$

As the diagram is drawn, the force Q is greater than P, and R is positive, or acts in direction A Q. But if P be greater than Q, R will be negative, or will act in a direction opposite to A Q, that is, in the direction of the force P.

Generally, $R =$ difference between P and Q, and acts in direction of the greater force. Also,

The greatest value of the resultant is the sum of the forces,
" least " " difference "

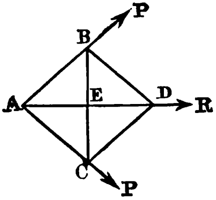
Further, as the angle between the forces is increased, the resultant is diminished.

(3). When $PAQ = 180°$, and $P = Q$, we have—

$$R = 0.$$

Or when two equal forces act in opposite directions on a point, they have no resultant, and therefore balance each other. It is only in this way that we can form the idea of equal forces, namely, as forces which balance when acting in opposite directions in the same straight line upon a free point.

(4). When $P = Q$, the parallelogram ABDC has four equal sides.



Hence in the triangles BAD, DAC we have $BA = AC$, AD common to the two triangles, and $BD = DC$.

Whence angle BAD = angle DAC.

This is also evident from the consideration that when $P = Q$ there is no reason which can be assigned for causing R to incline to P rather than to Q, which would not apply equally to causing it to incline to Q rather than to P.

Or when $P = Q$, the resultant bisects the angle PAQ.

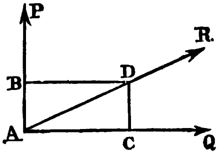
(5). Also when $P = Q$, the resultant may be found by a simple construction.

In this case, the two diagonals AD, BC intersect at right angles in E. Hence

$$R = AD = 2\,AE.$$

That is, if $AB = AC = P$, and we join BC and draw AE perpendicular to BC, we shall have

$$R = 2\,AE.$$



We will now examine two simple cases.

(1.) Let P and Q act at right angles.

Then $AD^2 = AC^2 + CD^2$,

or $R^2 = P^2 + Q^2$.

Cor. If $Q = P$, we have $R^2 = 2\,P^2$

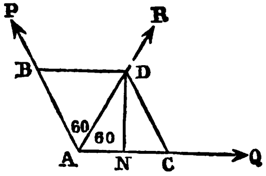
or $R = P\sqrt{2}$.

Now $\sqrt{2} = \sqrt{\frac{50}{25}} = \sqrt{\frac{49}{25}}$ very nearly.

$\therefore \sqrt{2} = \frac{7}{5}$ very nearly.

Whence $R = \frac{7}{5}\,P$.

(2.) Let $Q = P$, and let P and Q act at an angle of 120° Draw DN perpendicular to AC.



Then $R^2 = P^2 + P^2 + 2P \times CN$.

But since $P = Q$, the direction of R bisects the angle PAQ.

Hence $DAC = 60°$, and $AN = NC = \frac{AC}{2} = \frac{Q}{2} = \frac{P}{2}$.

$$\therefore R^2 = P^2 + P^2 + 2P \times \frac{P}{2} = P^2 + P^2 + P^2$$

Or $$R^2 = 3P^2$$

$$\therefore R = P\sqrt{3}.$$

Now

$$\sqrt{3} = \sqrt{\tfrac{48}{16}} = \sqrt{\tfrac{49}{16}} \text{ very nearly.}$$

$$\therefore \sqrt{3} = \tfrac{7}{4} \text{ very nearly.}$$

Or $$R = \tfrac{7}{4}P.$$

38. In the diagram (Art. 37) let R be the resultant of P and Q, which are respectively represented by AB, AC.

Draw DN perpendicular to AQ. Then, by Euclid:

$$AD^2 = AC^2 + CD^2 + 2AC \times CN,$$

$$\text{or,} \quad R^2 = P^2 + Q^2 + 2Q \times CN;$$

where CN must be determined by measurement or from tables.

We here remark:—

1. We may always remove a set of forces acting on a rigid body if we put their resultant in place of them.

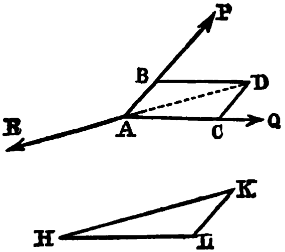
2. Forces which balance may be removed or supplied at pleasure.

3. If a flexible body be at rest under the action of forces we may suppose the power of motion taken away, and the particles to be rigidly connected.

4. If any point of a free body has no tendency to move we may suppose that point to be fixed.

5. When force acts upon a rigid body it may be supposed to be applied at any point in the line of its action.

39. **The triangle of forces.**—*Proposition*: If three forces acting at the same point balance each other, they are proportional to the sides of the triangle formed by any three straight lines parallel to their directions.



As before, let A B, A C completely represent P and Q; complete the parallelogram A B C D, then A D represents the resultant of P and Q. Let this resultant be R, then if D A be produced, a force R, equal to A D, and acting at A in D A produced will balance P and Q.

Hence the forces represented in magnitude and direction by A C, C D, D A will balance when applied to the point A.

Make a triangle H L K, whose sides are respectively parallel to the sides of the triangle A C D, viz., H L parallel to A C, L K parallel to C D, and K H parallel to D A.

Then this triangle will be similar to A C D, but it may be on a different scale; whereby if H L = 2 A C, we shall have L K = 2 C D, and K H = 2 A D, and so on. Hence the sides of the triangle H L K will represent the forces P, Q, R in magnitude and direction, the only change being in the unit of measurement, which is immaterial.

It follows that the three balancing forces are proportional to the sides of the triangle H L K, formed by any three lines drawn parallel to the forces.

Note, that each force is proportional to that side of the triangle to which it is parallel.

40. *Proposition*: If a triangle be formed by three lines at *right angles* to the directions of three forces which balance each other, the forces are respectively proportional to the sides of the triangle so formed.

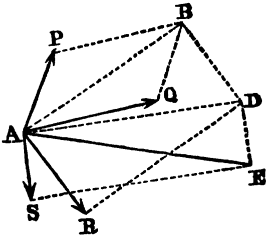
This is obvious, for the effect would only be to turn the triangle H L K round in its own plane through a right angle.

The propositions above stated embody the so-called principle of the *Triangle of Forces*, whereby the sides of a triangle represent the action of three balancing forces in magnitude and direction.

And it must be understood that this principle does not apply when the forces are parallel. For example, if P and Q were parallel forces, the point A would be removed to an infinite distance, and no triangle could be drawn. That case will be considered presently.

41. **Polygon of forces.**—*Proposition*: To find the resultant of any number of forces acting in one plane at the same point.

Let A P, A Q, A R, A S represent in magnitude and direction four forces acting on the point A.



Complete the parallelogram A P B Q, and join A B,
,, ,, A B D R, ,, A D,
,, ,, A D E S, ,, A E.

Then A B is the resultant of P and Q,
A D ,, A B and R, or of P, Q, R,
A E ,, A D and S, or of P, Q, R, S.

Hence A E represents in magnitude and direction the resultant of the forces P, Q, R, and S.

It is not necessary to draw all the lines in the figure; it is sufficient to commence by drawing P B parallel and equal to A Q, then to draw B D parallel and equal to A R, then to draw D E parallel and equal to A S, and then to join A E, which will be the required resultant.

The polygon A P B D is called the *polygon of forces*, and the proposition is merely an extension of the triangle of forces.

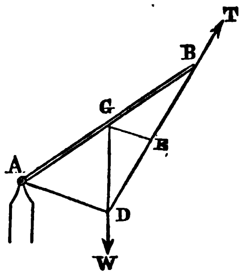
In drawing this polygon, if it turns out that E coincides with the point A, that shows that the forces so treated balance each other, or have no resultant.

42. **Concurrence of three balancing forces.**—Another principle of importance in the solution of problems may be thus stated :—

Proposition : When three forces act in one plane upon a body and two of them meet in a point, the third force must pass through the same point or there cannot be equilibrium.

This follows from the parallelogram of forces, for if two forces meet in a point they will have a resultant, and the third force which balances them can be no other than a force equal and opposite to this resultant, and passing through the point where the forces meet.

To apply this principle, suppose a heavy uniform beam A B to be hinged at A to a support, and to be sustained at an angle, as shown by a pull of a string B P fastened at B. Let it be required to find the pressure at the joint A, and the direction in which it acts.



Let P be the pressure at A, W be the weight of the beam acting at its centre G in the vertical line G W, and let T be the tension of the string acting in B T.

Produce T B to D, then the two forces T and W meet in D, whence the third force, namely, the pressure on the hinge, must also pass through D, or else there cannot be equilibrium.

Join A D and draw G E parallel to A D, then the triangle G E D is the triangle of forces in this case; for W acts in G D, T acts in D E, and G E is parallel to A D, which is the direction of the pressure at A.

Hence on a scale upon which G D represents W, we have P = G E, and T = D E.

Ex.: Let the beam be inclined at 30° to the horizon, and the string at 45° to the vertical, and let W = 100 lbs., find P and T. *Answer*: P = 111·8 lbs. and T = 167 lbs.

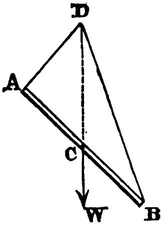
The principle may be further illustrated in the case of a uniform rod A B, suspended from a point D by two strings A D, B D, fastened in a knot at D, and of given length.

Since the rod is uniform, its weight W will act at the middle point C, and the forces in equilibrium are,

(1) the tension of A D

(2) " " B D

(3) the weight W in the vertical C W.



These three forces must meet in a point, hence the beam will rest with C vertically under D. It is also clear that, in a scale where W is represented by 2 D C, we have

tension of A D = A D

" D B = D B.

CHAPTER III.

THE LEVER, PARALLEL FORCES, COUPLES.

43. WE pass on to discuss the 'principle of the lever,' which may follow as a deduction from the parallelogram of forces, but which is more simply proved by a method dating back many hundred years.

Definition: A rigid body in which there is a fixed point or axis is called a *lever*, and the fixed point or axis about which the lever may turn, is called its *fulcrum*.

The name 'lever' is derived from a Latin word signifying to *raise* or *elevate*; it is commonly applied to any strong bar, such as a crow-bar, used for raising a weight or displacing an obstacle.

The fulcrum divides the lever into two parts called *arms*; when the arms are straight bars in the same line, we have a straight lever; when the arms meet at an angle, the instrument is called a bent lever.

44. *Proposition.*—To find the condition of equilibrium when unequal weights are hung at given distances from the fulcrum of a straight lever whose weight is neglected.

The proof is based on the observed fact that a heavy straight rod or bar, of uniform material and section, will balance when supported at its middle point.

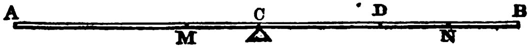
Let A B represent such a bar whose weight is P + Q, then A B will balance on a support or fulcrum C, placed under its middle point.

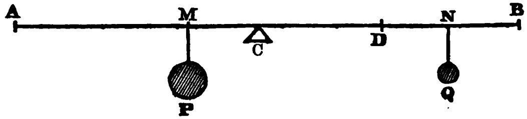
Divide A B in D, so that,

Weight of A D = P,
,, D B = Q.

Bisect A D, D B in M and N respectively ; then, since A D is a uniform rod, its weight may be supposed condensed into the centre M, and in like manner the weight of D B may be supposed collected at N.

Hence, if A B in the lower diagram be a rigid rod *without weight*, and weights P and Q be hung at M and N, this will not disturb the equilibrium, and the weights P and Q hanging on A B will replace the heavy uniform bar, and will balance on the point C.





What follows is geometrical reasoning.

$$\text{Since } CM = CA - AM = \tfrac{1}{2}AB - \tfrac{1}{2}AD = \tfrac{1}{2}DB,$$

$$\text{and } CN = CB - BN = \tfrac{1}{2}AB - \tfrac{1}{2}BD = \tfrac{1}{2}AD,$$

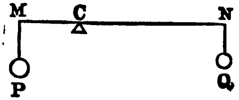
$$\therefore DB = 2\,CM,\ DA = 2\,CN.$$

$$\text{But} \qquad \frac{P}{Q} = \frac{DA}{DB}.$$

$$\therefore \frac{P}{Q} = \frac{2\,CN}{2\,CM} = \frac{CN}{CM}.$$

$$\therefore P \times CM = Q \times CN;$$

which is the condition of equilibrium.



Since, in the lower figure, A B is supposed to be without weight, we may now consider the actual lever to be the line M C N, the weight of which is neglected. On this supposition C M, C N will be arms, C being the fulcrum, and the weights P and Q will balance ; when,

$$P \times CM = Q \times CN.$$

Or, when $\frac{P}{Q} = \frac{CN}{CM}$.

Or, when $P : Q :: CN : CM$.

These are three different ways of stating the condition of equilibrium.

The converse is also true; that is, if

$$P \times CM = Q \times CN,$$

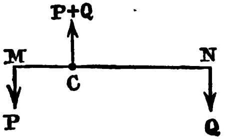
the weights P and Q will balance. It is possible to reason back by the same course reserved.

Cor. To find the pressure on the fulcrum.

It is evident that the whole weight of the rod A B rests upon the fulcrum, and that the pressure on the fulcrum is produced by the weight of the rod and by nothing else. Hence,

$$\text{Pressure on fulcrum} = P + Q.$$

45. It is now clear that the simple lever may be regarded as at rest under the action of three forces, viz., P at M, Q at N, and P + Q at C.



For, the pressure on C being P + Q, and action and reaction being equal and opposite, there would be equilibrium if we supplied at C a single force P + Q acting upwards.

If this be done the student will see that we may take indifferently either M, C, or N as the fulcrum of the lever.

Let M be the fulcrum, then

$$P \times CM = Q \times CN,$$

and

$$Q \times CM = Q \times CM,$$

$$\therefore (P+Q)\,CM = Q\,(CN+CM) = Q \times MN;$$

which gives the condition of equilibrium of a lever M C N under the forces P + Q at C, and Q at N, the point M being the fulcrum.

When M is the fulcrum the reaction at M supplies the downward force P; hence the pressure on M is also P, and acts upwards in a direction parallel to P+Q.

Or $$\text{pressure on fulcrum} = (P+Q) - Q$$
$$= P\,;$$

and acts in direction of the greater force, namely, P+Q.

If N be the fulcrum we have

$$(P+Q)\, CN = P \times MN,$$

and $$\text{pressure on fulcrum} = (P+Q) - P$$
$$= Q,$$

and acts upwards in direction of the greater force.

46. **Moment of a force.**—In examining the action of the lever, it is evident that P tends to turn the lever in one direction, and that Q tends to turn it in the opposite direction, and these turning effects balance each other. We infer from the above proof that the product

$$P \times \text{arm } CM$$

measures the turning effect of P acting at an arm CM. Thus, a weight 6 at an arm 2 will balance a weight 3 at an arm 4, for $6 \times 2 = 12$, and $3 \times 4 = 12$.

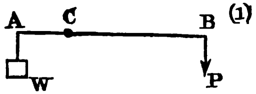
It is next to be observed that so long as P acts perpendicularly to the arm CM, the product $P \times CM$ remains the same whatever be the direction of CM, whether horizontal or inclined to the vertical at any given angle. It is also the same when P acts vertically upwards.

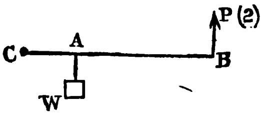
Since everything depends on this product $P \times CM$, it is convenient to give it a name, and we call it the *moment* of P round the fulcrum C. The word 'moment' has a technical meaning, and indicates the effect of P as an agent to turn the lever.

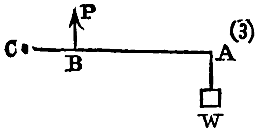
If P lbs. acts at arm of 2 feet, the moment of P is 2 P, and so on. If P lbs. acts at an arm of *a* feet, the moment of P is P *a*.

47. **Levers of different kinds.**—There are three different kinds of lever, marked (1) (2) (3) in the diagram, the weight of A B being neglected.

In (1) let P act downwards at B and balance W at A, then $P \times CB = W \times CA$.







A crowbar employed to raise a stone is a lever of the *first* kind. A pair of scissors is a double lever of this kind, the joint being the common fulcrum of the two levers, and material to be cut the resistance.

In (2) let W and P both act at the same side of the fulcrum, W being nearer to C, the condition of equilibrium will be that the moments are equal, or that $W \times CA = P \times CB$.

The oar of a boat is a lever of the *second* kind, the blade of the oar in the water being the fulcrum, and the resistance being the pressure on the rowlock.

A wheel-barrow is another example. The centre of the wheel is the fulcrum, and the weight W acts at the centre of gravity of the load.

A pair of nut-crackers is a double lever of the second kind.

In this case P must be *less* than W.

In (3) let W and P act on the same side of C, but let P be nearer to C than W. As before $P \times CB = W \times CA$.

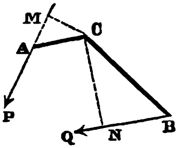
The fore-arm of a man is a lever of the *third* kind. The elbow-joint is the fulcrum, the weight W is anything placed in the hand, and the contraction of a muscle in the arm exerts an upward pull at a point near the elbow-joint.

A pair of tongs is a double lever of the same kind.

Here P must be *greater* than W.

In (1) the pressure on the fulcrum is $P + W$ acting downwards.
„ (2) „ „ „ $W - P$ „
„ (3) „ „ „ $P - W$ „ upwards.

48. **Bent lever.**—In a bent lever of the most general kind, the arms are inclined at any angle, and the forces act at given angles to the arms.



It has been shown that if CM be the arm of a lever, and a force P acts at M in a direction perpendicular to CM, the turning effect or moment of P is P × CM.

In like manner the turning effect of Q acting perpendicularly to an arm CN would be Q × CN.

If P × CM = Q × CN there will be equilibrium, whatever be the positions of CM, CN, as in the case of the bent lever MCN, where P and Q act at right angles to the respective arms CM, CN.

Draw CA, CB inclined at any angle to CM, CN, and let ACM, BCN represent triangles with rigid sides. Then P may act at A, and Q at B without disturbing the equilibrium Hence the forces P and Q acting on the arms of the bent lever ACB will balance when P × CM = Q × CN.

That is, when two forces P and Q act at any angles on the arms of a bent lever ACB, and CM, CN are drawn from the fulcrum C, perpendicular to AP, BQ respectively, there is equilibrium when P × CM = Q × CN.

49. **Moments of any number of forces.**—The product, P × CM, is the *moment* of P acting on the arm CA, at an angle CAP. The term moment may be defined with greater generality than in Art. 46.

Definition: The moment of a force is the product of the force into its perpendicular distance from the centre of motion.

It is now clear that if any number of forces act upon a body to turn it in any direction about a given axis, the turning effect of each force is represented by its moment, and the whole turning effect is the sum of the moments of all the forces.

The moment of a force being *positive* when the tendency to turn is in one direction, it would be *negative* when the tendency to turn is in the opposite direction.

Hence we add or subtract moments just as we add or subtract any other magnitudes of a like kind. Since any number of forces acting on a body in one plane may have a resultant which produces the same effect as their combined action, it is evident that the moment of the resultant of any number of forces about a point is equal to the sum of the moments of the respective forces about the same point.

50. The principle of the lever may be stated generally as follows :—

Proposition : If any number of forces acting on a rigid body in one plane tend to turn it about a fixed axis, there will be equilibrium when the sum of the moments of the forces tending to turn the body in one direction is equal to the sum of the moments of the forces tending to turn it in the opposite direction.

51. **Parallel forces.**—The composition and resolution of parallel forces may be deduced from the parallelogram of forces by employing various artifices, but we prefer to deal with this branch of the subject as an application of the principle of the lever.

Proposition : To find the resultant of two parallel forces acting in the same direction on a rigid body, and the point where it acts.

Let P and Q be two parallel forces acting in parallel directions MP, NQ, upon a rigid body.

Draw MCN cutting MP, NQ, at right angles, and let MCN be supposed rigid. (Diagram on next page.)

Then MCN is a lever, and there must be some point C in MN, such that $P \times CM = Q \times CN$.

To find C, let $CM = x$, $MN = a$.

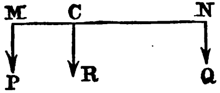
Then $$Px = Q(a - x) = Qa - Qx.$$

$$(P + Q)x = Qa$$

$$x = \frac{Qa}{P + Q} \qquad (1)$$

Also the pressure on the fulcrum is $P + Q$.

Hence a single force $P + Q$ acting at C is produced by the combined action of P at M, and Q at N, and is therefore their resultant. That is



$$R = P + Q,$$

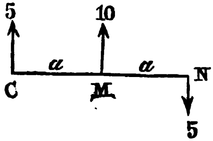
and R acts at the point C, which is given by equation (1).

This proposition is perfectly general, and if one of the forces, as P, acts upwards, we have only to make P negative, and we shall have

$$R = Q - P,\ x = \frac{Qa}{Q - P}.$$

If P be greater than Q, we shall have both R and x negative.

Ex.: Let $P = 10$ lbs., and act upwards,
$Q = 5$ lbs., and act downwards.



Then $R = 5 - 10 = -5$,

$$x = \frac{5a}{5 - 10} = -\frac{5a}{5} = -a.$$

Hence C is at a distance a on the left hand of M, also $R = 5$, and acts upwards.

Definition: The point C is called the centre of the parallel forces P and Q.

It is, in fact, the point at which the resultant of P and Q may be supposed to act.

52. **Couples.**—When P and Q act in opposite directions, and Q is equal to P, we have,

$$R = P - P = 0,$$

or a system of two equal and opposite parallel forces has no resultant.

But although such a system has no resultant, in the sense that it cannot be represented by a single force, it is a powerful mechanical agent which is continually met with.

Definition : A pair of equal and opposite parallel forces is called a *couple.*

Definition : The perpendicular distances between the forces is called the *arm* of the couple.

Definition : The product of either force into the arm is the *moment* of the couple.

Let A P, B P, be the directions of two parallel forces P and P acting on a rigid body, and constituting a couple.

Let any straight line A B cut A P, B P at right angles.

Let E be any point in A B, and suppose A B to be rigid, then the turning effect of the forces P, P about the point E is

$$P \times AE + P \times BE, \text{ or } P \times AB.$$

Again, let F be any point in B A produced, then the turning effect about $F = P \times FB - P \times FA$,

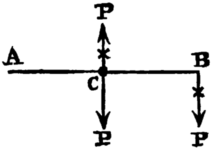
$$= P \times AB, \text{ as before.}$$

Also, A B is any straight line in the plane of the forces ; hence we infer that the action of the couple tends to turn the body about any line round which it can turn, provided only that this line is perpendicular to the plane in which the forces act, and further, that the measure of this tendency to turn is the product of P into the arm of the couple, or is $P \times AB$, which we have agreed to speak of as the 'moment of the couple.'

53. In the drawing of a screw-press the student will find an example of the use of a couple. The bar or handle which turns the screw is grasped by the two hands, one at each end of the bar. In order to turn the screw there would be a pull with one hand and a push with the other ; these two forces would be nearly equal, and the turning effect would be the product of either force into the distance between the parts grasped by the hands. Of course this distance is not susceptible of accurate measurement.

If only one hand were used, and a hard pull exerted, the tendency would be to upset the apparatus, and that is one reason why a double handle is employed.

Thus, let A B be the handle, C its centre, P a pulling force exerted *only* at B. Supply at P two equal and opposite forces P, P; this will not affect the action which is taking place. Then the two forces C P, B P (marked with a cross in order to distinguish them) form a couple whose moment is P × C B, which is just one half what it was before; and there is left the force C P, which, acting horizontally towards the person who is pulling at B, tends either to upset the apparatus or to carry it off the table on which it is placed.



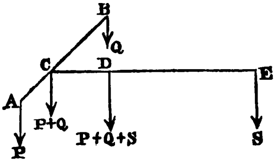
It is to be noted that when any number of couples act at the same time to turn a body about the same axis, the effect of the combined action of the couples is represented by adding together their respective moments, the negative sign being prefixed to any moments which tend to turn the body in the opposite direction to that agreed upon as being positive.

54. **Resultant of parallel forces.**—The method of finding the resultant of two parallel forces may be extended to any number of forces.

Proposition: To find the resultant of any number of parallel forces.

Let P, Q, S represent three parallel forces acting on a rigid body.

Let C be the centre of the forces P and Q, found as in Art. 51, then P + Q is the resultant of P and Q, and acts at the point C given by the equation



$$AC\,(P + Q) = Q \times AB.$$

By a like construction, D is the centre of (P + Q) and S, and is given by the equation

$$CD\,(P + Q + S) = S \times CE.$$

Also the resultant of (P + Q) and S is

P + Q + S, acting at the point D.

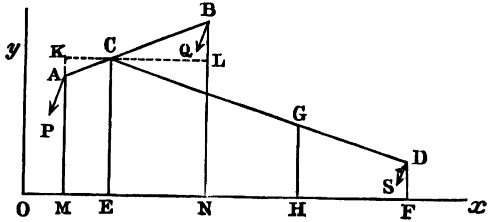
And the like for four, five, or any greater number of parallel forces.

Hence the resultant of any number of parallel forces is the sum of the forces, the negative sign being prefixed to any forces which act in the opposite direction to that assumed as being positive.

55. **Centre of parallel forces.**—The method of co-ordinates affords a more easy and practical method of finding the position of the centre of any number of parallel forces, the term *centre* being applied to the point where the resultant of the forces acts.

It is proposed in the first place to ascertain the centre of three parallel forces by the method of co-ordinates, and the general law which applies in any case will then be apparent.

Proposition: To find the centre of any number of parallel forces acting in one plane on a rigid body.



Let P, Q, S be parallel forces acting at the points A, B, D of a rigid body. Take O x, O y rectangular axes in the plane of the forces; draw A M, B N perpendicular to O x. Let C be the centre of the forces P and Q, found by Art. 51, and draw C E perpendicular to O x. Also draw K C L parallel to O x, meeting the directions of P, Q in K and L.

Then
$$\mathrm{P\times AM=P\,(KM-AK)=P\times CE-P\times AK},$$
$$\mathrm{Q\times BN=Q\,(LN+BL)=Q\times CE+Q\times BL}.$$
$$\therefore\ \mathrm{P\times AM+Q\times BN=(P+Q)\,CE+Q\times BL-P\times AK}.$$

But by similar triangles we have $\mathrm{\frac{BL}{AK}=\frac{CB}{CA}}$,

whence $\mathrm{Q\times BL-P\times AK=AK\,(Q\times\frac{BL}{AK}-P)}$,
$$\mathrm{=AK\,(Q\times\frac{CB}{CA}-P)},$$
$$\mathrm{=\frac{AK}{CA}\,(Q\times CB-P\times CA)}.$$

But $\mathrm{Q\times CB-P\times CA=0}$, $\therefore\ \mathrm{Q\times BL-P\times AK=0}$,
whence $\mathrm{P\times AM+Q\times BN=(P+Q)\,CE}$.

In like manner, if G be the centre of the forces (P+Q) acting at C and S acting at D, and GH, DF be drawn perpendicular to Ox, we shall have
$$\mathrm{(P+Q)\times CE+S\times DF=(P+Q+S)\,GH},$$
or
$$\mathrm{P\times AM+Q\times BN+S\times DF=(P+Q+S)\,GH},$$

whence the following rule, which clearly applies to any number of forces, viz.:

Multiply each force into its perpendicular distance from a given line, as Ox, then the sum of such products will be equal to the product of the sum of the forces into the perpendicular distance of their *centre* from the same line.

The magnitudes of the forces and their points of application being known, the distance of the centre from Ox can be determined, and in like manner, by dropping perpendiculars on Oy and proceeding exactly in the same manner with reference to that line, the distance of the centre from Oy can be determined, and its exact position in the plane xOy can be ascertained, for we now have determined the two rectangular co-ordinates of the centre of parallel forces.

It is apparent that the position of the centre is quite independent of the directions in which the forces act.

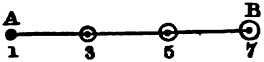
CHAPTER IV.

THE CENTRE OF GRAVITY.

56. WHEN the parallel forces are the weights of the separate parts of a body, the centre of the forces in action is the centre of gravity of the body.

Definition.—The centre of gravity of a body is that point in which the whole weight may be supposed to act, or is the centre of parallel forces due to the weights of the respective parts of the body.

Ex. Bodies weighing 1, 3, 5, 7 lbs. are placed at equal distances of 1 foot along a given line A B: find their centre of gravity.



Here the centre of gravity is in the line A B; let x be its distance from A, then

$$(1 + 3 + 5 + 7)\,x = 1 \times 0 + 3 \times 1 + 5 \times 2 + 7 \times 3$$

$$\text{or } 16\,x = 34,$$

$$x = \tfrac{34}{16} = \tfrac{17}{8} = 2\tfrac{1}{8} \text{ feet from A.}$$

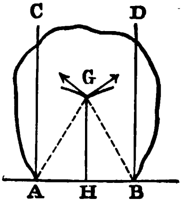
57. **Descending tendency of the centre of gravity.**—It follows from the above definition, that if the centre of gravity of a body be a fixed point, and if gravity be the only force acting, the body will be at rest in all positions; but we have further to determine what will happen when the centre of gravity is not a fixed point, although at rest for the time.

Here we remark that, inasmuch as the centre of gravity is continually drawn downwards by the force of gravity, it always tends to fall or come to a lower level, but never to rise.

Hence we infer that the centre of gravity will not remain permanently at rest, if it can begin to move by descending, and that it will not alter its position by gravity alone if it is compelled to rise or to move in a horizontal line.

58. **Equilibrium of a body on a horizontal plane.**—This principle of the descending tendency of the centre of gravity gives an easy method of determining whether a body will stand or fall when placed on a horizontal plane. All that is necessary to be done is to set out a figure and determine from geometrical conditions whether the centre of gravity will begin to move, if it move at all, by ascending or descending.

Thus let A B C D be a vertical section through G, the centre of gravity of a body resting on a horizontal plane A B. With radii A G, G B draw small circular arcs, as shown, and draw the verticals A C, B D.

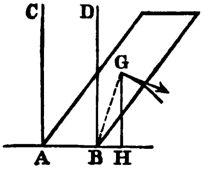


So long as G lies within the space C A B D, both these arcs point upwards, as indicated by the arrows which are at right angles to A G, B G respectively; or the centre of gravity can only move by ascending, and therefore, according to the principle stated, it will not move at all.

Draw G H, a vertical through G, cutting A B in H. Then if H lies between A and B, the body will stand firmly on the horizontal plane, so far as motion in the plane of the paper is concerned.

In order to test fully the equilibrium of a body, vertical planes must be drawn in other directions through G, but the same reasoning applies, and the body will stand so long as the vertical through the centre of gravity falls within the base. If, however, the point G lies without the space C A B D, let it lie beyond B D, as in the diagram.

Proceeding as before, it is apparent that G will begin to move about B by descending, as shown by the arrow drawn at right angles to B G, and there is nothing to prevent this motion, or the body will fall over by turning on the point B. The like would be true if the centre of gravity were on the left of A C.



Hence, a body will not stand upon a horizontal plane if the vertical through its centre of gravity falls without the base. It is also apparent that when the centre G lies either in A C or B D, it tends to move in a horizontal line, and the body will not fall, although it may be easily disturbed.

For simplicity, we have considered the case of a body resting on a horizontal plane, but the same reasoning applies when a body rests on an inclined plane, the nature of the surfaces in contact being such that no sliding or slipping takes place. The conclusion is perfectly general, and the body will not fall over so long as the vertical through its centre of gravity falls within the base.

59. **Stable and unstable equilibrium.**—The student will now be able to understand fully the distinction between *stable* and *unstable* equilibrium.

A body is in stable equilibrium if, when it is slightly disturbed from its position of rest, it comes back again to that position, and is in unstable equilibrium when, under like circumstances, it moves away altogether from that position.

The general law may be stated as follows :—

The equilibrium of a body is stable or unstable, according as its centre of gravity is in its highest or lowest position.

Where a body is at rest with the centre of gravity in its lowest position, the equilibrium can be no other than stable, for any disturbance must raise that centre, whereupon the force of gravity will bring it down again to its former position.

Also, a body always rests in stable equilibrium when its centre of gravity lies below the point on which it is supported. In this case any disturbance would raise the centre, whereupon gravity would bring it back again to the old position.

But if the centre of gravity be in its highest position, and the body be slightly disturbed from the position of rest, the centre has thereby fallen to a lower level. The action of gravity is to bring this centre lower and not to raise it, and therefore the body will move away from the position of rest. In such a case equilibrium is unstable.

As a matter of fact, a body never does rest in a position of unstable equilibrium. There is theoretically a position of rest, but practically it is unattainable. It is the old problem of making an egg stand upon one end on a plate.

There is an intermediate case of which examples are common enough—namely, where, after a slight disturbance from the position of rest, the body has no tendency to move, either back to its original position or away from it.

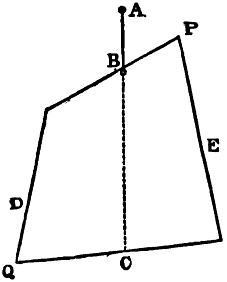
In such a case the equilibrium is *neutral*, and the test is, either that the centre of gravity does not move at all, or that it moves in a horizontal line.

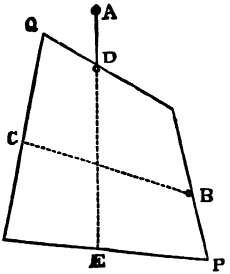
The first case occurs where a body is suspended on a line passing through its centre of gravity, whereas an example of the second case would be seen where a cylinder or a sphere lies upon a horizontal plane. Any position is one of equilibrium, the cylinder or the ball will not roll if merely shaken a little.

60. **Examples.**—The centre of gravity of a body may be found in a few simple cases by geometry, of which examples will be given. The centre of gravity of a plane figure of any shape, such as a piece of cardboard, or the like, may be found by experiment.

It has been seen that, when a body hangs suspended from a point, the centre of gravity lies vertically below the point of suspension.

Thus let a piece of flat board, P Q, be suspended from A





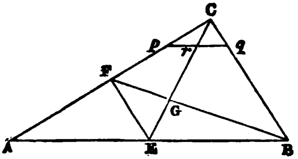
by a string A B, then the centre of gravity of the board lies in the vertical A B C.

Again suspend the board by the string A D, then the centre of gravity lies in the vertical A D E.

It follows that, if we mark with a pencil on the board the two directions B C, D E, the centre of gravity will be their point of intersection, namely, G.

61. *Proposition* : To find the centre of gravity or plane triangle.

Let A B C be a triangle, bisect A B in E, join C E, and draw *p r q* parallel to A B and meeting C E in *r*.



Then $pr : AE$
$:: Cr : CE$
$:: rq : EB.$

But $AE = EB \therefore pr = rq$, or *p r q* is bisected in *r*, and the like is true of every other line drawn parallel to A B, wherefore the centre of gravity of the triangle lies somewhere in C E.

Again bisect A C in E, and join B G, E F. Then, by like reasoning, the centre of gravity of the triangle lies in B F, and therefore G is the centre of gravity of the triangle.

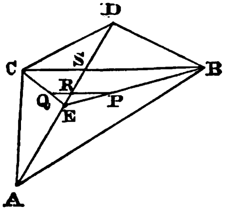
By similar triangles FEG, CGB, we have

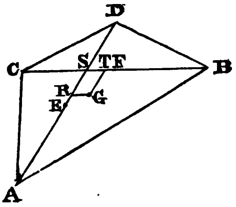
$$\begin{aligned} EG : GC &:: FE : CB \\ &:: AE : AB \\ &:: 1 : 2 \\ \therefore GC &= 2\,GE, \\ \text{or } EG &= \tfrac{1}{3}\,EC. \end{aligned}$$

That is, if a straight line be drawn from one angular point of a triangle to the centre of the opposite side, the centre of gravity of the triangle lies in this line at a distance from the angular point equal to two-thirds the length of the line.

62. To find the centre of gravity of a quadrilateral.

Let ABCD be a quadrilateral; join AD, BC, intersecting in S; bisect AD in E, join BE, CE, take $EP = \frac{1}{3}EB$, and $EQ = \frac{1}{3}EC$; and join PQ, cutting AD in R.





Then P is the centre of gravity of ABD;
Q " " " ACD;

$\therefore$ the centre of gravity of the whole figure lies in PQ.

Also $EP : EB :: 1 : 3 :: EQ : EC$;

$\therefore$ PQ is parallel to CB,

and $ER = \frac{1}{3}ES$.

Hence we have this construction. Draw the diagonals AD, BC. Bisect AD in E, take $ER = \frac{1}{3}ES$, and draw RG parallel to CB; the centre of gravity of ABDC lies somewhere in RG.

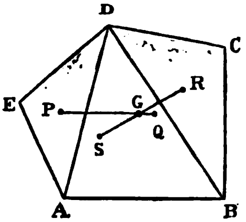
By like reasoning bisect CB in F, take $FT = \frac{1}{3}FS$, draw

T G parallel to A D, then the centre of gravity of A B D C lies somewhere in T G.

Therefore the required centre of gravity lies in G, the point of intersection of T G and R G.

63. To find the centre of gravity of a pentagon.

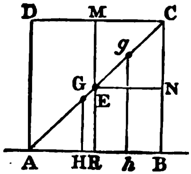
Let A B C D E be a pentagon, join A D, B D, and take P, the centre of gravity of triangle A E D, and Q the centre of gravity of quadrilateral A D C B, and join P Q. Thus the centre of gravity of the whole figure lies in P Q. Again, take R, the centre of gravity of triangle B D C, and S the centre of gravity of quadrilateral A E D B, and join R S.



Then the centre of gravity of the whole figure lies in S R; ∴ the point G, in which P Q, S R intersect, is the centre of gravity of the pentagon.

64. From a square A B D C, whose centre is E, the square M E C N is cut away. Find the centre of gravity of the remainder.

Join A C, then the point required must lie in A C. Let it be G, and draw G H perpendicular to A B. Also let g be the centre of M N, and draw $g h$, E R perpendicular to A B.



Then, by property of the centre of gravity,

$$ABCD \times ER = MN \times gh$$
$$+ MAN \times GH.$$

Let $AB = a$, then $ABCD = a^2$, $MN = \frac{a^2}{4}$,

$$MAN = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}, gh = \frac{a}{2} + \frac{a}{4} = \frac{3a}{4};$$

$$\therefore a^2 \times \frac{a}{2} = \frac{a^2}{4} \times \frac{3a}{4} + \frac{3a^2}{4} \times GH,$$

or $$8a = 3a + 12\,\mathrm{GH},$$
or $$12\,\mathrm{GH} = 5a,$$
$$\mathrm{GH} = \frac{5a}{12}.$$

In like manner $\mathrm{AH} = \frac{5a}{12}$, whence G is known. It is, therefore, apparent that $\mathrm{AG} = \frac{5}{12}\,\mathrm{AC}$.

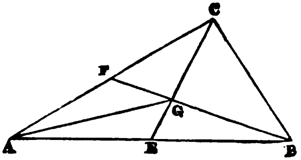
This may be proved in a different manner.

Thus $\mathrm{ABCD} \times \mathrm{AE} = \mathrm{MN} \times \mathrm{A}g + \mathrm{MAN} \times \mathrm{AG}$,

or $$a^2 \times \frac{1}{2}\,\mathrm{AC} = \frac{a^2}{4} \times \frac{3}{4}\,\mathrm{AC} + \frac{3a^2}{4} \times \mathrm{AG},$$
or $$8\,\mathrm{AC} = 3\,\mathrm{AC} + 12\,\mathrm{AG},$$
whence $$\mathrm{AG} = \frac{5}{12} \times \mathrm{AC}.$$

65. We conclude this chapter with a proposition to be found in most books on mechanics, namely :—

If G be the centre of gravity of a triangle A B C, the forces represented in magnitude and direction by the lines G A, G B, G C will keep the point G at rest.



The proof is the following :—

Since G A is equivalent to G E, E A, and G B is equivalent to G E, E B, therefore G A, G B are equivalent to A E, E B and 2 G E.

But A E = E B ∴ the forces A E, E B balance ∴ G A, G B are equivalent to 2 G E.

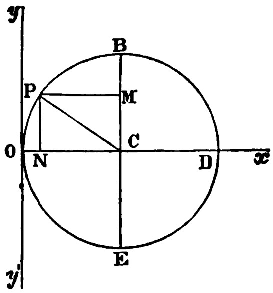
But it has been proved that G C = 2 G E ∴ the forces G A, G B, G C are in equilibrium when acting on the point G.

CHAPTER V.

THE CONVERSION OF MOTION.

66. It is now time to begin the study of circular motion, from which so many of the movements in machinery are derived, and our first object will be to show that the circular motion of a point is capable of being resolved into two simple straight line motions.

To prove this we apply the system of co-ordinates (Art. 25). Let P be a point moving uniformly in a circle whose centre is C; O C D, B C E two diameters at right angles; O x, O y being rectangular axes. Join P C, and draw P N, P M perpendicular to O C, C B respectively.



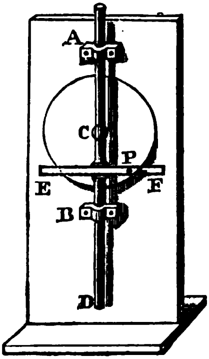
As stated in Art. 30, the point N is called the *projection* of P on O C, and as P travels from O to B, N travels from O to C.

In like manner, M is the projection of P on B C, and travels from C to B, while P describes the curve O B.

Hence, if a point P receives at the same time the motion of N along O x and that of M parallel to O y, it will describe the circular arc O P. That is, circular motion is made up of two simple motions, viz. of the points N and M, and it may be resolved into the components O N and N P.

Circular may be resolved into reciprocating motion by suppressing one of these components and retaining the other.

67. Let us, for example, suppress the motion parallel to O y, and retain only the motion of N. This may be done by the contrivance shown in the diagram, which represents a model used in the normal school. A pin P fastened to a circular disc, is rotated round C by a handle at the back of the model. A horizontal slotted bar E F is attached to a vertical bar, A B D, moving in guides at A and B.



As P rotates, the motion along E F is rendered inoperative or is suppressed, and the other motion, viz. that of the point N, is retained and imparted directly to the sliding bar A D.

Thus the circular motion of P is resolved into a reciprocating motion, and the sliding bar A D copies exactly the motion of the point N.

As P goes round in a complete circle (see diagram, p. 58) N moves to and fro along the diameter O C D. The motion of P being uniform, that of N is variable. The point N begins its motion from rest, attains its greatest velocity when it comes to C; after passing C its velocity becomes less until the point D is reached, and N commences the return motion.

68. The resolution of circular into reciprocating motion in the manner pointed out can be easily rendered apparent by placing a silvered bead on one side of the rim of a small wheel mounted on an axis.

If the wheel be set spinning with its face towards the observer, the bead will describe a circle, whereas when looked at edgeways, the bead will appear to move to and fro in a line, which may be horizontal or vertical, according to circumstances. Thus the motion of N or M may be realised.

The term *harmonic motion* has been applied to indicate the motion of N, because the end of the prong of a vibrating tuning-fork has this motion when sounding a note.

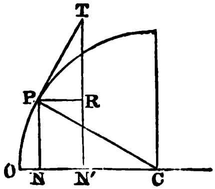
There is no space to discuss the subject fully, but we mention the term because it is now in common use.

Also when N is at C, M is at B, and, taken as a whole, the motion of M is identical with that of N.

The student should note that the points M and N, each of which has a separate harmonic motion, do not start together. On the contrary, M is in the middle of its swing when N is just beginning to move. Circular motion is only possible when the two simple motions are properly timed, one must have a start of half a swing before the other. If both M and N started together, P would describe a straight line and not a circle.

69. To find the velocity of N.

Let PT represent the velocity of P at any instant, and draw PN, TN′ perpendicular to OC. Also draw PR perpendicular to TN′.

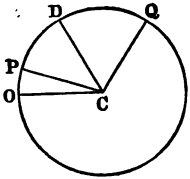


Then NN′ is the resolved part of PT along OC, whence

$$\frac{\text{vel. of N}}{\text{vel. of P}} = \frac{NN'}{PT} = \frac{PR}{PT} = \frac{PN}{PC}.$$

From which equation the velocity of N is known in terms of that of P.

70. **Measurement of angular velocity.**—Before going further it is necessary to agree upon a method of measuring circular motion.



For this purpose, let the diagram represent a circle whose centre is C. Let OC be a fixed line, P and Q any two points in the circumference, and D a point such that arc OD = radius OC. Join PC, DC, and QC. Then

$$\frac{\text{angle OCP}}{\text{angle OCD}} = \frac{OP}{OD} = \frac{OP}{OC} = \frac{\text{arc}}{\text{radius}}$$

If we select as a unit the angle O C D, which it will be quite proper to do, inasmuch as it is an invariable magnitude, we have O C D = 1, and

$$\text{OCP} = \frac{\text{OP}}{\text{OC}} = \frac{\text{arc}}{\text{radius}}.$$

We thus obtain a simple measure of the magnitude of an angle, for taking the vertex of the angle O C P as a centre, and drawing a circle of any radius, as in the figure, we express the magnitude of the angle O C P by the ratio $\frac{\text{OP}}{\text{OC}}$.

Definition: The angle subtended at the centre of a circle by an arc equal to the radius is the unit of circular measure.

When an angle is expressed in terms of this unit, it is said to be expressed in *circular measure.*

It is easy to see that

$$\text{angle OCD} = \frac{180}{\pi} = \frac{180}{3{\cdot}1416} = 57{\cdot}29577 \text{ degrees.}$$

71. It is now competent for us to estimate the velocity of a point moving in a circle from a new point of view.

Referring to Art. 70, let P describe the circle O D Q with a uniform velocity v.

Then v is the arc described by P in one second. But it is clear that while P is describing a given arc, the radius C P is describing a given angle, and that the motion of P is shown by the angle which C P describes in one second.

Take C O as a fixed line of reference, then the *rate of change* of the angle O C P is called the *angular velocity* of C P.

Angular velocity may be uniform or variable; and if it be *uniform*, it is measured by the angle described in a unit of time.

The unit of time is always one second, unless the contrary be expressed.

It is the universal practice to appropriate the Greek letter ω (*omega*) for expressing the unit of angular velocity.

Following this notation, let O P be the arc described by P in one second, when moving uniformly in the circle O P Q.

Let v = the velocity of P.

r = radius of the circle.

Then

$$\text{OCP} = \frac{\text{arc}}{\text{radius}} = \frac{\text{OP}}{\text{OC}}$$

or

$$\omega = \frac{v}{r},$$

whence

$$v = \omega r,$$

which is the equation connecting the uniform linear velocity of P with the uniform angular velocity of C P.

When variable, the angular velocity of C P is measured by the angle which *would be* described in a unit of time, if C P retained throughout that unit the motion it has at the instant considered.

72. **Transfer of circular motion.**—In dealing with circular motion one of two things may be done.

1. We may draw the circular motion from the circumference of the circle, as we should draw off a piece of string from a reel on which it has been wound.

2. We may suppress one of the two components of the circular motion, and may take the other in the form of a simple harmonic motion.

Each method leads to a variety of useful mechanisms. The first is carried out in toothed wheels and in the numerous pulleys with bands or straps running on them which may be seen in a workshop, while the second exercises the skill of the mechanician in devising combinations where these movements play an important part.

73. We proceed to examine the method of transferring circular motion from one rotating disc to another.

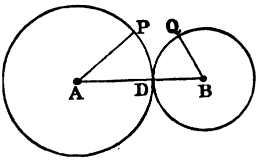
This may be done by actual rolling contact.

Let there be two rolling discs, A and B, turning on parallel axes, and having their edges in contact.

It is evident that if A and B were properly constructed and very accurately adjusted, it would be quite possible for A to move B by friction alone, the two discs rolling smoothly and evenly upon each other without any slipping of the surfaces in contact. But we could not expect A to overcome any great resistance to motion in B, or, in other words, we could not in practice convey any considerable amount of force by the action of one disc upon the other.

74. *Proposition* : When two circles roll together, their uniform angular velocities are inversely as the radii of the circles.

Let two circles, centred at A and B, move by rolling contact through the corresponding angles PAD and DBQ.



Let $AD = a$, $DB = b$.

Then $PAD = \frac{PD}{a}$, $QBD = \frac{QD}{b}$,

and $PD = QD$

But
$$\frac{\text{angular vel. of circle A}}{\text{angular vel. of circle B}} = \frac{PAD}{QBD}$$
$$= \frac{PD}{a} \div \frac{QD}{b}$$
$$= \frac{b}{a},$$

which proves the proposition.

Let A make a complete revolution, then $2\pi a$ is the length of the circumference of A, and $2\pi b$ is the length of the circumference of B.

Hence, if a be less than b, B will rotate through an angle which is to four right angles as

$$2\pi a : 2\pi b.$$

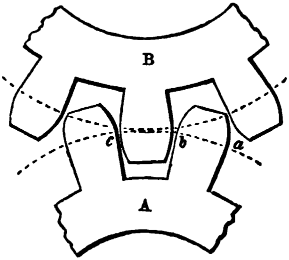
or B will make a fraction of a revolution given by

$$\frac{2\pi a}{2\pi b} \text{ or } \frac{a}{b};$$

That is, when A makes one revolution,

B makes $\frac{a}{b}$ of a revolution.

75. **Toothed wheels.**—The transmission of force being an essential condition in machinery, the discs A and B are provided with teeth, as shown, the two dotted circles representing two imaginary circles, called *pitch circles*, which, if the teeth be properly made, will roll truly upon one another during the motion.



This matter should be well understood, the actual projections come in contact at different points, and are perpetually rubbing and grinding upon each other. We cannot prevent their doing so, but we can apply certain geometrical rules for shaping the teeth whereby we shall provide that, whatever may be the action of the individual teeth, the pitch circles will roll upon each other without any slipping or irregular action.

Definition: The pitch circles of two wheels are two circles which represent, by rolling in contact, the actual motion of the wheels when working together.

Ex.: Suppose that the two axes, at a distance of 16 inches, are to be connected by wheelwork, and are required to rotate with velocities in the proportion of 3 to 5. Two circles, centred upon the respective axes, and having radii 6 and 10 inches, would, by rolling contact, move with the desired relative velocity, and would be the pitch circles of the wheels in question. Hence, the pitch circles are determined in the first instance, and in a mechanical drawing where combinations of wheelwork occur, it is usual to represent wheels by their pitch circles.

When the number of teeth upon a wheel is indefinitely increased, the wheel itself degenerates into the pitch circle.

Definition: The pitch of a tooth is the space *a c* upon the pitch circle cut off by corresponding edges of two consecutive teeth.

So much of the tooth as lies within the pitch circle is called its *root* or *flank*, and the part ouside the pitch circle is called the *point* or *addendum*.

Let n be the number of teeth upon a wheel,

D the diameter of pitch circle, P the pitch of a tooth, then,

$n\ \mathrm{P}$ = circumference of pitch circle = $\pi\ \mathrm{D}$.

It follows that the numbers of teeth in a set of wheels in gear will be directly as the diameters of the pitch circles of the respective wheels.

The wheels, of which a portion is shown in the diagram, p. 64, are called spur wheels, and are those in which the teeth project radially along the circumference.

In a *face wheel*, cogs or pins, acting as teeth, are fastened perpendicularly to the plane of the wheel.

In a *crown wheel* the teeth are cut on the edge of a circular band.

An *annular wheel* has its teeth formed upon the inside of an annulus or ring instead of upon its outer circumference.

A *rack* is a straight bar provided with teeth, and a wheel with a small number of teeth is called a *pinion*.

Gearing and *gear* are words used to indicate the combination of any number of parts in a machine working together for a common object.

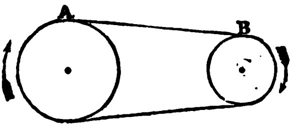
Toothed wheels are said to be *in gear* when they are capable of moving each other, and *out of gear* when they are shifted into a position where the teeth cease to act.

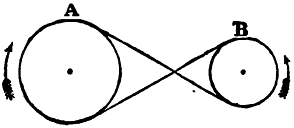
The term *axis*, in its strict meaning, denotes the central line of a cylinder, and is a mathematical phrase. An engineer distinguishes a heavy cylindrical piece of metal as *shafting* or a *shaft*, and applies the term *spindles* to smaller cylindrical bars ; a wheelwright speaks of the *axle* of a wheel as the piece on which it turns, whereas a watchmaker discards the term axle and calls the same thing an *arbor*.

The axle of a wheel is commonly a cylindrical bar running completely through the wheel, and supported at both ends on cylindrical hollow bearings which fit the shaft and envelope it closely, any motion endways being prevented by collars or shoulders upon the shaft.

Of two moving pieces that which transmits motion is termed the *driver*, and that which receives it is the *follower.*

Belts or bands.—Circular motion may be transferred from one rotating disc to another by means of belts or cords, otherwise called *bands.*





The principle of this most common method of transferring motion, is shown in the diagram where A and B represent two circular discs at a small distance from each other, connected by an endless cord which does not slip upon the discs. This cord or band is a common tangent to each disc, and the two parts may or may not cross as shown. It is apparent that the direction of rotation of B will be the same as that of A when the tangents do not cross, and will be opposite to that of A when they do cross.

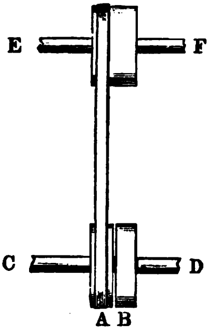
Thus, A may drive B in the same or in opposite directions at pleasure. Also the velocity of B may be varied by altering the relative sizes of the two discs.

For example, let diameter of A = 12 inches, and diameter of B = 9 inches. Since the band is not supposed to slip, the motion of B is the same as if it rolled directly upon it.

Hence $$\frac{\text{angular vel. of A}}{\text{angular vel. of B}} = \frac{9}{12} = \frac{3}{4}$$

Or if A makes 60 revolutions in a given time, B will make $\frac{4}{3} \times 60$ or 80 revolutions in the same time.

Fast and loose pulleys.—In practice the discs of which we have spoken are pulleys, and where the band is a flat strap or belt, as is common when machinery is driven by steam-power, the contrivance known as *fast and loose pulleys* is introduced. Such pulleys are shown in the diagram, and marked A and B; they are made of iron, with a flat rim on which the band runs. The fast pulley A is keyed to the shaft C D, and B rides loose upon it.



E F is the driving shaft to which a drum is keyed, and the object of the contrivance is to impart motion to C D at pleasure, the driver E F being kept continually rotating by the steam engine or other motor. The pulley A being keyed upon the shaft, and B running loose, the shaft C D will rotate when the strap is on A, but will stop when the strap is shifted to B, and the machinery connected with C D will remain at rest also.

This is a most useful contrivance, but to render it effective two things are necessary.

1. There must be a ready means of shifting the strap from A to B. This is done by a forked lever, between the prongs of which the strap runs. A little consideration will show that in order to shift the strap it must be pressed by the fork on the *advancing* side, and not after it has left the pulley. The strap adheres by friction to the surface, and the fork must act on the part which is coming up to the pulley in order to carry it on one side laterally.

2. The strap must not slip off. To prevent this, especially when the pulley runs at a high speed, the rim is made a little rounded or convex. It is an experimental fact that if a tight strap or belt be running upon a cone, it will continually tend to mount up the slant side and to get as near to the base of the cone as possible. This is the opposite to what one might expect before trying it, and arises pro-

bably from the fact that the belt, which is somewhat elastic, tends to lie flat on the slant surface of the cone, whereby the advancing edge is continually brought a little nearer to the base, and so runs up instead of slipping off. Hence in a convex pulley, the strap keeps on the top of the convex portion, which is the middle of the pulley.

In light machinery round cords, commonly made of catgut, are employed. They are called bands, and are provided with a hook and eye to make them continuous. They run in grooved pulleys, as a round cord has no tendency to run out of a groove. There is no ready mode of shifting them, as in the case of flat straps.

76. **Guide pulleys.**—When it is desired to shift the direction of a band from one line A B to another line C D, intersecting it at any angle, a *guide pulley* is used. This is simply a pulley so placed that the plane of the disc of the pulley lies in the plane in which the lines A B, C D are situated. In that case the advancing side of the band must always remain in the plane of the guide pulley, and the band has no tendency to run off.

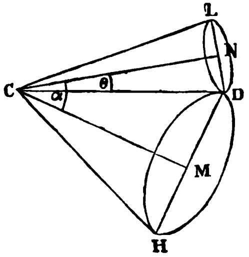
If the lines A B, C D do not intersect, we can always draw a line, E F, intersecting them at some points, E and F, and can insert one guide pulley at E, having its plane coincident with the plane A E F, and can introduce a second guide pulley at F having its plane coincident with the plane E F D, and thus the cord will run securely upon the two guide pulleys and carry on the motion from the line A B to the line C D.

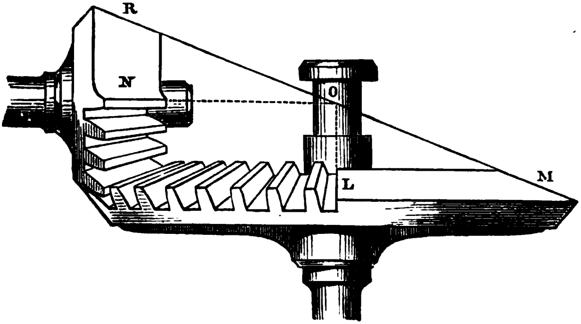
Bevel wheels.—The spur wheels, already described, are suited to convey motion only between parallel axes. It often happens, however, that the axes concerned in any movement are not parallel, and as a consequence they may, or may not, meet in a point. If the axes do not intersect, we proceed by successive steps and continually introduce intermediate intersecting axes, and thus we are led to the use of inclined wheels whose axes meet each other, and which are known as *bevel wheels*.

In order to understand the principle on which bevel wheels are constructed, we proceed to inquire into the rolling action which is possible between two cones.

The cones with which we are concerned are *right cones*, such as are shown in the diagram; where, for example, the cone C D H is produced by the rotation of the right-angled triangle C D M round the axis C M, the slant side C D sweeping out the slant surface of a cone having a circular base of radius, M D.

A little consideration, or, what is better for beginners, an experimental trial with two small cones properly mounted, will show that *two right cones* which have a *common vertex*, such as those sketched in the diagram, will roll together without any sliding or slipping of the surfaces in contact.





Hence, if two portions of right cones, such as L N, N R, have a common vertex, O, they will roll together, and will communicate motion from the axis O L to the axis O N.

The surfaces L M, N R are called *pitch cones*, and just as, in the case of spur wheels, we formed teeth upon a pair of pitch circles capable of imparting motion between two parallel axes, so, in the case now before us, we may form teeth upon a pair of pitch cones as indicated in the diagram, and the teeth so formed will, if rightly made, communicate a smooth and uniform motion from one axis, O L, to another, O N, inclined to it at a given angle, which in this case is a right angle.

If the student will carefully examine the action of two bevel wheels in gear, he will always be able to trace the existence of the ideal cones to which we have referred, and will find the position of the common vertex in the imaginary line of intersection of the axes of the two pieces of shafting on which the respective bevel wheels are fixed.

Equal bevel wheels, whose axes are at right angles, are technically known as *mitre wheels.*

The wheels here spoken of are ordinary bevel wheels, which are so commonly used in machinery, but there are other forms of bevel wheels, known as skew bevels, from the twisted appearance of their teeth, to which the rules above stated do not apply. There is not space here to discuss the subject of skew bevels.

CHAPTER VI.

PRINCIPLE OF WORK—FRICTION.

77. *Work* is done when a weight is raised in opposition to the force of gravity.

This is the simplest conception we can form of what is meant by work, and is that from which the measure of work is derived.

Unless the contrary be stated, the weight raised is measured in pounds, and the height to which it is lifted is measured in feet.

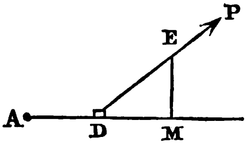
Definition : The unit of work is the work done in lifting one pound through a height of one foot, and is called a *foot-pound.*

The student will readily understand that this unit is properly selected. Thus, in raising 1 lb. through 1 foot we do a certain amount of work, and in raising 2 lbs. through 1 foot we do twice as much work, and so on. Again, if we raise 1 lb. through 2 feet we do twice as much work as if we raised it through 1 foot, and so on.

Hence by the rule-of-three principle work is estimated by the product of the number of pounds raised into the height through which it is raised. Thus the work done in raising 10 lbs. through a height of 5 feet is 5 × 10 or 50 foot-pounds.

78. *Generally*, work is done by a force, when some resistance is continually overcome, whereby the point of application of the force is continually moved notwithstanding the resistance.

Thus, let D E represent completely a force P acting at D, and let the force move the point D from A to D against some resistance.



Draw E M perpendicular to A D produced, then D M is the resolved part of P along A D.

And work done by P = A D × D M, where A D is expressed in feet, and D M „ „ pounds, the answer being a given number of foot-pounds.

If P acted perpendicularly to A D, it would do *no work* while D was moved along A D, but the work done would be due to the action of some other force not before the student.

79. *Definition* : The term *power* is applied to the rate of doing work.

Watt estimated that the sustained work of a horse continued for one minute would raise 33,000 lbs. through one foot in one minute, or that 33,000 foot-pounds represented the rate of work of a horse per minute.

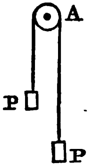
The number 33,000 foot-pounds is the *unit* of horse power, and is employed for measuring the work of a steam-engine which continues to act minute after minute. If an engine is working at 10 horse power, it will, by acting continuously at a uniform rate, perform in each minute the work of 10 × 33,000 foot-pounds.

The rate at which a man can do work per minute is taken at 3,300 foot-pounds, or $\frac{1}{10}$ that of a horse, but it varies considerably.

80. **The principle of work.**—We come now to the *principle of work*, which is of the highest importance in solving mechanical problems, and in order to form an idea of what it means, let us take a simple case.

Conceive that there is before us a delicate balance, carefully poised, and with equal weights in the respective scale pans. It would be found that either end of the beam would rise or fall on the slightest touch ; that is to say, we should find there was no work to be done in lifting one scale pan while the other went down. The forces in action balance among themselves, and oppose no resistance to any external force, such as the supposed touch of the finger on the beam of the balance.

Or suspend two equal weights P, P, on a fine string passing over a light wheel A, which has a grooved rim and can turn very easily on a support through its centre.



In this case, if one of the weights be raised one inch, the other will fall one inch, and the system will be in equilibrium after the movement just as it was before it. The weights are equal, and pull in opposite ways at equal distances from the centre of the wheel. We can readily see that a single force may do work, but we cannot conceive the possibility that two equal and opposite forces can, under these circumstances, do anything but counteract each other. The principle of work expresses this fact in general terms, and is the following:

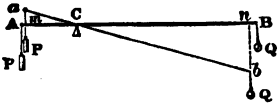
Proposition: If a system of bodies be at rest under the action of any forces, and be moved a very little, no work will be done.

Conversely: If no work is done during this small movement, the forces are in equilibrium.

No elementary proof of this general proposition has yet been invented. It is, however, quite in accordance with all experience that a system of balancing forces should be regarded as incapable of doing work. Also if work be done when a weight is raised, then work is undone when the same or an equal weight is let down to the original level. It is as if money were paid into a bank in the one case and drawn out in the other. In the system of two equal weights hanging over a pulley the work done in raising P through an inch on the one side is immediately lost or expended by allowing P to descend an inch on the other side.

Hence work done may be *positive* or *negative.* Work is positive when a weight is raised, and negative when it is allowed to descend. Or, generally, referring to Art. 78, positive work is done when D is moved from A to D, and negative work is done when D is moved back again from D to A. In order that D may return to A the force P must be overpowered, and the work previously done is lost.

81. As an example we refer to a lever A C B, in which a weight P hung at A balances a weight Q hung at B. For simplicity, suppose the lever to be without weight, or else that one arm is loaded so that it balances on the fulcrum C when P and Q are removed. Tilt the lever through a small angle into the position acb, and let the directions of P and Q cut A C B in m and n respectively.



Then the point at which P acts describes a vertical space ma,
and „ „ Q „ „ „ nb.

$$\text{Therefore work done on P} = \text{P} \times ma$$

$$\text{„ „ by Q} = \text{Q} \times nb,$$

$$\text{and these are equal, } \therefore \text{P} \times ma = \text{Q} \times nb$$

$$\text{or } \frac{\text{P}}{\text{Q}} = \frac{nb}{ma} = \frac{\text{C}b}{\text{C}a} = \frac{\text{CB}}{\text{CA}},$$

which is the well-known condition of equilibrium on the lever.

82. **The laws of friction.**—All bodies are rough more or less, and when placed in contact with each other they exert a resistance to sliding motion which is called *friction.* The French writers on mechanics call friction a *passive resistance*, because it is only brought into play when an attempt is made to move one body over another.

Friction is of great utility in some cases, and in others it is a source of waste and expense. Without friction an arch would not stand, a nail or a screw would be useless, and a railway train could not leave a station, but in parts of machinery where pieces are revolving, friction is a direct loss of power, and the object of the mechanician is to reduce it as much as possible.

Up to the present time we have only considered the lever as a mechanical instrument, and have taken no notice of the resistance to motion which takes place at its point of support, viz. the fulcrum, which is often a cylindrical pin working in a cylindrical support or bearing.

In the discussion of the machines and parts of machines which is to follow, no notice whatever is taken of the resistance of friction, inasmuch as there is not space to enter into this subject fully, and the student could not master the difficulties to be encountered until after some preliminary training in mathematics.

We proceed to state the laws of friction (as they are termed) when one body rests upon another, and remark that these laws are only approximately true, and present nothing more than a general idea of the action of frictional resistance.

(1.) The resistance of friction acts in the opposite direction to that in which the body slides or tends to slide.

(2.) The resistance of friction, when the body is on the point of sliding, is always a certain fraction of the pressure between the surfaces in contact. This fraction is called the *coefficient of friction.*

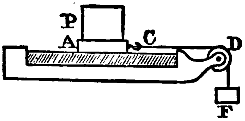
It is usual to denote this coefficient by the Greek letter μ (corresponding to *m*), whereby, if P be the pressure between two surfaces, the resistance of friction $= \mu$ P.

It is stated that—

$\mu =$ ·08 for metals on metals with oil.
$\mu =$ ·17 for metals without oil.
$\mu =$ ·33 for wood on wood.
$\mu =$ ·65 for dry masonry.

(3.) When the force tending to make the body slide is less than this fraction of the pressure, the body will remain at rest; when it is greater, the body will slide if it be shaken or disturbed a little so as to overcome the adhesion which comes into play when two bodies rest in contact. This adhesion somewhat increases the effect of friction.

Laws 2 and 3 may be shown approximately by experiment as follows:—On a firm horizontal table there is placed a slab of the material selected, say cast iron, and upon it is placed another small slab, A, of the material to be tested in relation thereto. The slab A is attached to a weight by a fine silk cord passing over a pulley at D, as shown.



The slab is loaded with different weights, and the amount of the hanging weight, F, which will just cause A to slide, is tested, the table being tapped to avoid the effect of adhesion.

(4.) The resistance of friction is independent of the extent of the surfaces in contact.

This follows from the law that the friction is a certain fraction of the pressure between the surfaces.

Thus let a body of weight, W, rest upon two flat portions A and B, and let P be the pressure on A, Q the pressure on B. Then

$$P + Q = W.$$

But friction at A $= \mu\,P$, friction at B $= \mu\,Q$.

Therefore, total friction $= \mu P + \mu Q$

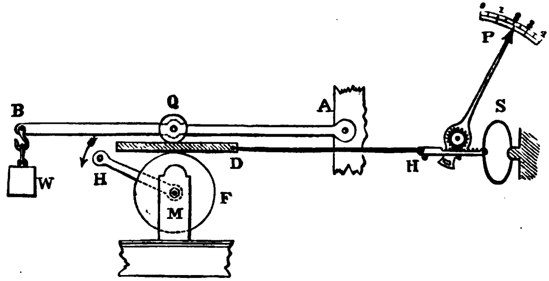
$$= \mu\,(P + Q)$$

$$= \mu\,W.$$

Again, if there were three supports, the total friction would still be μ W, which is quite independent of the amount of surface in contact.

Note.—This law only applies when flat surfaces are pressed together by a force which is external to the bodies. It does not apply to the case of the friction of a piston moving in the steam cylinder of an engine, or to the case of water moving along a pipe.

(5.) The friction is independent of the velocity when there is motion.



The drawing shows an apparatus belonging to the Normal School, with which this law may be illustrated.

The surfaces, whose frictional resistance, when rubbing under pressure at different velocities, is to be tested, are the flat piece, D, and the cylinder, F.

The piece, D, is connected by a cord to a small rack, H, and a spring, S. The rack gears with a pinion attached to a pointer, P, the end of which travels over a graduated scale.

Above the sliding piece is a lever, A B, having a fulcrum at A and a small roller at Q. To the end, B, are hung different weights, such as W, which press the roller, Q, upon the sliding piece, D, and thus cause D to press upon F. Inasmuch as Q turns very easily on its axis, there is no sensible resistance to the sliding of D upon F by reason of the contact and pressure of the lever, A B. If Q were merely a fixed projection, the friction between it and D would obviously vitiate the result.

The method of experimenting with the model is as follows:

Remove W, when the weight of A B will cause a slight pressure between the lower surface of D and that of the roller, F. On turning the handle, H M, in the direction of the arrow, the piece, D, will tend to move towards the left and exert a slight pull on the string, D H. The graduated scale is so set that the index-finger, P, now points to zero.

Next let a weight of (say) 7 lbs. be hung at B, when there will be a much increased pressure on Q, due to the leverage of the arm, B Q A, and the pressure between D and F being increased, there will be a proportionate increase in the friction, whereby, on turning the handle, the pull on the string, D H, will be increased, and the index-finger, P, will move a certain distance along the scale. Again, suspend 14 lbs. at B and turn the handle, when it will be found that the index-finger travels through twice the distance along the scale. This exhibits the law that the friction is proportional to the pressure.

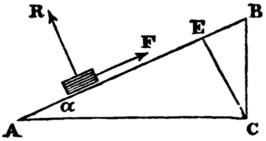
But, further, the apparatus shows that the friction is independent of the velocity when there is motion, for with any given weight hung at B, the index, P, remains very nearly in the same position on the scale whether the handle be turned slowly, or as rapidly as we please.

Experiments which have recently been made at the Institution of Mechanical Engineers appear to show that when a loaded shaft is rotating in its bearing the friction increases with an increase of velocity after a surface speed of 100 to 150 feet per minute has been obtained.

83. **The angle of friction,** often called 'the angle of repose,' is an important quantity in studying the laws of friction, as we shall now endeavour to make clear. It has been stated that the coefficient of friction is the ratio of F to W, where F is the friction, and W the mutual pressure between the surfaces. We proceed to show that the coefficient of friction may be found experimentally by a distinct method founded on the principle of the inclined plane.

If a body with a rough surface be placed upon a horizontal plane which is provided with a hinge so as to be capable of being set at different angles to the horizon, it will be found that there is some particular inclination, dependent on the surfaces in contact, at which the body will begin to slide when somewhat shaken or disturbed.

The diagram represents a body so placed, the perpendicular pressure between the body and the plane being R, and the friction being F. Draw C E perpendicular to A B, then,



$$\frac{F}{R} = \frac{BE}{CE} = \frac{BC}{AC}.$$

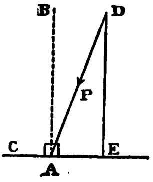
Hence the coefficient of friction $= \frac{BC}{AC}$, and this ratio is known when B A C is determined. It has been agreed to call B A C the angle of repose or the angle of friction, and when B A C is known we can by construction deduce the coefficient of friction.

Ex. : Let B A C = 10°, then coefficient of friction = ·18.

84. **The angle of resistance.**—It has been explained that, when a body rests upon a plane, the pressure which it exerts on the plane is called *action*, and the support which it receives from the plane is called *reaction*, and these are equal and opposite.

The student will understand that if a surface be perfectly smooth it can offer no resistance in any direction except in a line perpendicular to itself, or that the reaction of a smooth surface must always be felt in a line perpendicular to the surface.

But with a rough body the case is different. It is matter of common experience, that if a rough body, A (whose weight is neglected), rests on a plane, C E, and a pressure, P, be exerted upon A in an oblique direction, the body will not begin to slide until the direction of P has been tilted through a considerable angle, such as B A D.



Let D A represent P in magnitude and direction, draw D E perpendicular to the plane, then

D E represents the pressure on the plane
E A ,, ,, force tending to produce motion.

So long as E A is counteracted by friction the body will not move.

But the greatest value of the resistance of friction
= (coefficient of friction) × D E.

Hence the body will not slide until
A E = (coefficient of friction) × D E.
or until A D E = angle of friction.
or until B A D = angle of friction.

The angle B A D is called by many writers *the angle of resistance*, and thus the same angle appears under different names according to the point of view from which it is regarded.

The student should bear in mind that the body, A, will slide on the plane, C E, as soon as the oblique pressure acts at an angle greater than the angle of friction. He should also remember that this conclusion is based on the assumption that the weight of the body, A, is so small that it may be neglected in comparison with the oblique pressure, otherwise the problem becomes a little more complicated, for it may frequently happen that the friction set up by the weight of a body may be sufficient to counteract the effect of any oblique pressure, even when acting at an angle much exceeding the angle of friction.

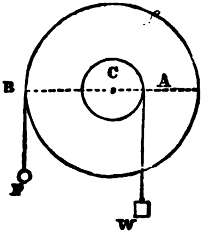
CHAPTER VII.

SIMPLE MACHINES.

85. **The wheel and axle.**—The wheel and axle is a form of lever which allows a weight to be raised through any given height. It is a practical device for continuing the action of a lever as long as may be required, the weight rising all the time.

The wheel and axle is familiar to every one; it is used for drawing a bucket out of a well. The rope is wound round an axle, and the power is exerted on a lever handle. Sometimes the wheel has handles, as in the steering-wheel of a vessel. The capstan is an example of the wheel and axle, the wheel being replaced by a set of capstan-bars.

In the sketch, the large circle represents the wheel and the small circle represents the axle. The weight W hangs on a cord wound round the axle, and the power P is a weight hanging on another cord wound round the wheel. The wheel and axle have a common axis, C.



It is evident that A C B is a lever, C being the fulcrum, and the condition of equilibrium is

$$\frac{P}{W} = \frac{CA}{CB} = \frac{\text{radius of axle}}{\text{radius of wheel}}.$$

86. The same result follows from the principle of work. Let the wheel make one revolution, then

[illegible] lever with
[illegible] is a lever
[illegible] *ey-block* in
[illegible] two metal
[illegible] cylindrical
[illegible] umber of
[illegible], and the
[illegible] a double
[illegible] on. The
[illegible] block in
[illegible] ion.

[illegible] ample, we
[illegible] on string
[illegible] at A to a
[illegible] ance being
[illegible] the same
[illegible] o the sys-
[illegible] e suppose
[illegible] e points B

[illegible] A [illegible] P,
[illegible] P.

[illegible] ant of the
[illegible] P,

[illegible] e of work.
[illegible] ortened by
[illegible] the string
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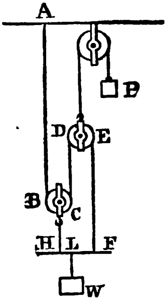
$$\therefore\quad \text{W} \times 1 = \text{work done on W.}$$
$$\text{P} \times 2 = \text{work done by P.}$$

and these are equal, since P balances W.

$$\therefore\quad \text{W} = 2\,\text{P}.$$

89. **Different systems of pulleys.**—Questions relating to pulleys may be readily solved by the consideration that wherever a string leaves a pulley we may supply a force equal to the tension of the string.

This method will be apparent from the following example, which is a combination put together as an illustration. It is of no practical use, inasmuch as it will turn out to be less effective than a single movable pulley.



Here tension of AB = tension of DC = tension of EF.

But tension of DC

$$+\quad \text{tension of EF} = \text{P};$$
$$\therefore\quad \text{tension of EF} = \frac{\text{P}}{2};$$
$$\therefore\quad \text{tension of HC} = \text{P};$$
$$\therefore\quad \text{W} = \text{P} + \frac{\text{P}}{2} = \frac{3\,\text{P}}{2}.$$

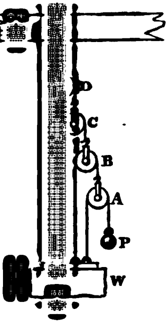
90. The *first system of pulleys* is shown in the left hand diagram on the next page. Here each pulley hangs by a separate string, whereby

Pull on AB = 2 P,
" BC = 4 P,
" CD = 8 P,
" ED = 16 P,
∴ W = 32 P;

or, generally, $\text{W} = 2^n\,\text{P}$ where n is the number of movable pulleys.

In the *second system* the same string winds round each [illegible] wer block,

[illegible] P.



[illegible] agram,

[illegible] al number

[illegible] hould con- [illegible] ent of the [illegible] to make [illegible] ple of the [illegible] the lower [illegible] block B.

[illegible] s attached [illegible]

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[illegible] given [illegible] truly [illegible] ained [illegible] seen [illegible] trans- [illegible] rm of [illegible] of the [illegible] cles at [illegible]

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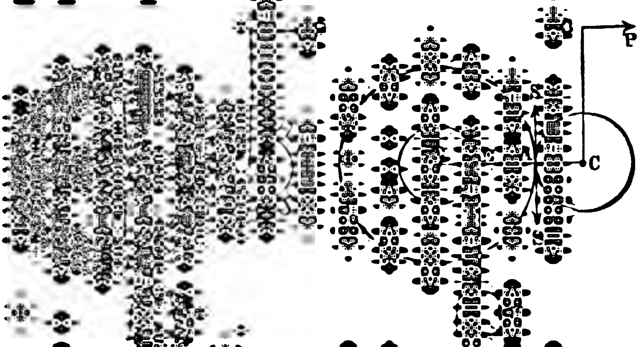


[illegible] dle C B [illegible] drum [illegible]

[illegible] B, and [illegible] w, the [illegible]

The right-hand figure shows the wheels replaced by their pitch circles, where a force, A S, acting upwards at right angles to C A, is the action of C on H, and a force, A S, acting downwards at right angles to C A, is the reaction of the wheel H on C.

Then B C A forms a bent lever,

$$\text{and} \quad P \times CB = S \times CA.$$

Also A E D is a straight lever, D being the centre of the wheel H,

$$\text{and} \quad S \times AD = W \times DE$$

$$\text{whence,} \quad P \times S \times CB \times AD = W \times S \times CA \times DE,$$

$$\text{or} \quad P \times CB \times AD = W \times CA \times DE.$$

$$\text{or} \quad \frac{P}{W} = \frac{CA \times DE}{CB \times AD}.$$

Let m be the number of teeth on C, and n the number of teeth on H, then by Art. 74.

$$\frac{CA}{AD} = \frac{m}{n}$$

$$\therefore \frac{P}{W} = \frac{m}{n} \times \frac{DE}{BC}.$$

Ex.: Let $m = 20, n = 100, \frac{DE}{BC} = \frac{1}{2}$

$$\text{Then} \quad \frac{P}{W} = \frac{20}{100} \times \frac{1}{2} = \frac{1}{10}$$

$$\text{or} \quad W = 10\,P.$$

92. Applying the principle of work, we proceed as follows :—

Let C make one complete revolution, then

P describes a space $= 2\pi \times CB$.

But by Art. 74, when the wheel C makes one turn, the wheel D makes the fraction $\frac{CA}{AD}$ of one turn, and therefore the weight W rises through a space.

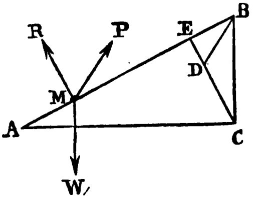
$$= \frac{CA}{AD} \times 2\pi \times DE.$$

Therefore $$\mathrm{P} \times 2\pi \times \mathrm{CB} = \mathrm{W} \times \frac{\mathrm{CA}}{\mathrm{AD}} \times 2\pi \times \mathrm{DE}.$$

$$\therefore \quad \frac{\mathrm{P}}{\mathrm{W}} = \frac{\mathrm{CA} \times \mathrm{DE}}{\mathrm{CB} \times \mathrm{AD}};$$

the same result as before.

93. **The inclined plane** is a plane inclined at any angle to a horizontal plane, and our object is to find the force, P, which is competent to support a weight, W, on an inclined plane by acting upwards at a given angle, as in the sketch.



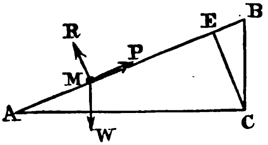
Let A B be the plane; draw A C, B C, horizontal and vertical lines, through A and B. Let M be a body, of weight W, supported by a force, P, acting in a plane passing through A B C. Let R be the reaction of the plane which is perpendicular to A B. The forces R, P, W are indicated in the sketch. Take B C to represent W, and draw C E perpendicular to A B, and B D parallel to M P. Then B D C is the triangle of forces, whereby

B C represents W in magnitude and direction,
D B ,, P ,, ,,
C D ,, R ,, ,,

Whence the relation between P, W, and R is known.

Cor. 1. Let P act parallel to the plane, then B D becomes B E, and E B C is the triangle of forces.

Hence $\frac{\mathrm{P}}{\mathrm{W}} = \frac{\mathrm{BE}}{\mathrm{BC}}$.



But $\frac{\mathrm{BE}}{\mathrm{BC}} = \frac{\mathrm{BC}}{\mathrm{AB}}$ by reason that the triangles E B C, A B C are similar;

$$\therefore \quad \frac{\mathrm{P}}{\mathrm{W}} = \frac{\mathrm{BC}}{\mathrm{AB}} = \frac{\text{height}}{\text{length}}.$$

Also $\frac{R}{W} = \frac{EC}{BC} = \frac{AC}{AB} = \frac{\text{base}}{\text{length}}$.

94. The same result follows from the principle of work for if W be raised from A to B,

$$\text{work done on } W = W \times BC,$$
$$\text{,, by } P = P \times AB.$$

And these are equal;

$$\therefore \quad P \times AB = W \times BC,$$
$$\text{or} \quad \frac{P}{W} = \frac{BC}{AB} = \frac{\text{height}}{\text{length}}.$$

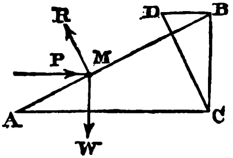
Ex.: If $BC = 1$, $AB = 10$, then $W = 10\,P$, or a force of 10 lbs. will support a weight of 100 lbs. on the plane.

Cor. 2. Let P act horizontally, then BD is horizontal, and

$$\frac{P}{W} = \frac{BD}{BC} = \frac{BC}{AC} = \frac{\text{height}}{\text{base}}.$$

Also by principle of work—

$$\text{work done on } W = W \times BC$$
$$\text{,, by } P = P \times AC$$
$$\therefore \quad P \times AC = W \times BC.$$

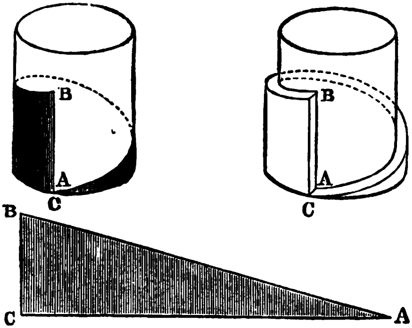


95. **The screw.**—When a triangle, ABC, forming an inclined plane is wrapped round a cylinder, as shown, the extreme point, A, of the triangle being made to coincide with C, we have a *screw-thread.*

To get an idea of this take a cylinder of wood, such as a common roller or a pencil, cut a piece of paper into the shape of a right-angled triangle, ABC, and blacken the edge, AB. Then fasten the side, BC, to the cylinder lengthways with gum or drawing-pins, and wrap the paper round the cylinder tightly. The blackened edge will form a curve running round the cylinder at a constant inclination to its axis. This curve represents the *thread* of a screw.

If BC be of sufficient length the thread will make several turns, the paper overlapping, and in that way the experiment is not quite accurate, for the cylinder is continually getting larger, which ought not to happen. But as the only object is to form an idea of a screw-thread, this is not of importance.

In order that the screw-thread may be useful as a mechanical instrument it must present a considerable surface upon which force can act, and for this purpose suppose the



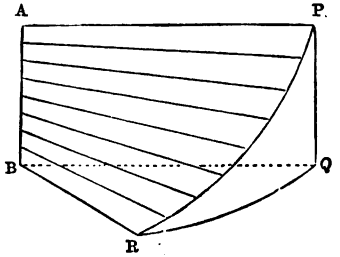
inclined plane to be a plate of some thickness, as shown, and to be wrapped round the cylinder, we shall then have a ridge encircling it, and the upper surface of this ridge will form a screw-thread in common use. When the triangle, A B C, wraps exactly once round the cylinder, the line B C is the *pitch* of the screw-thread.

Also the angle B A C is the *inclination* or angle of the screw-thread.

96. It is common to call a screw-thread a *spiral* thread. Thus we speak of a spiral staircase, a structure often seen in old castles or cathedrals as formed of stone, and in more modern buildings there are light spiral staircases of iron. It is instructive to examine the manner in which such a staircase is constructed, for a rough model of a screw surface may readily be made on the same plan.

Definition : If a horizontal line, A P, which passes through a fixed vertical line, A B, be caused to revolve *uniformly* in one direction while the end A moves uniformly along A B, it will trace out a *screw surface.*

The different positions of A P are shown in the diagram, and may be imitated by threading a number of light laths of wood on an iron spindle supported on a stand. In the figure P R Q is part of a cylinder whose axis is A B, and whose diameter is 2 A P Also P R Q would unwrap into a right-angled triangle forming an inclined plane. The surface is shown in perspective, and A P = B R. Also P R is a screw-thread on the cylinder of radius A P.



Definition : The *pitch* of a screw is the space along A B through which A P moves in one revolution.

Definition : The line A B is the length of the screw surface A B P R, and the angle P R Q is the angle of the screw-thread.

In the diagram A P is describing a *right-handed* screw. If it revolved in the opposite direction while descending it would describe a left-handed screw. As a rule, all ordinary screws are right-handed. Thus, in screwing down the lid of a box the screw-driver is turned to the right to insert the screw, and to the left to bring it out. Whereas, if the screw were left-handed we should turn it to the left hand to insert it, and to the right hand to bring it out.

Also the student will observe that the sketch in Art. 95 shows a right-handed screw, and that if the triangle B C A were wound in the opposite direction round the cylinder the screw would be left-handed.

97. The screw-thread used in machinery is a projecting rim of a certain definite form running round a cylinder,

[illegible] red to

[illegible] re the [illegible] oughly

[illegible] prac- [illegible] n inch

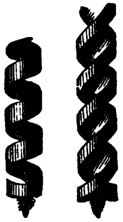
[illegible]

[illegible] screw- [illegible] ngular- [illegible] ny, the [illegible] ade by

[illegible] rposes, [illegible] ve less [illegible] half the [illegible] d if the [illegible] space

[illegible] ound a [illegible] should [illegible] threads were wound side by side we should obtain a *double-threaded* screw. The object of increasing the number of threads is to fill up the space which would be unoccupied if a fine thread of rapid pitch were traced upon a bolt, and thus to give the bolt greater strength in resisting any strain which tends to strip away the thread. Thus a shutter bolt which is required to enter rapidly with a few turns is commonly a double or treble-threaded screw.

Increasing the number of threads makes no difference in the pitch of the screw, which is dependent on any one continuous thread of the combination. The drawing shows a single and double threaded screw as applied to a screw auger for boring holes in wood. The difference is at once apparent to the eye, and the point of the tool shows another use of the screw, viz. for feeding on the cutter. The small-pointed screw continually advances into the material and takes the rotating cutter with it, the shavings being pushed along the hollow spiral groove between the threads.



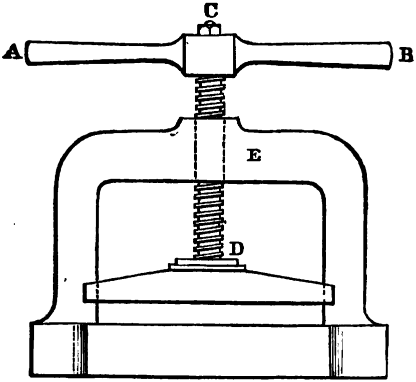
99. **The screw-press.**—In order to apply the screw as a mechanical instrument it must be provided with a nut or casing, inside which there is hollowed out a groove which is the exact counterpart of the ridge or thread upon the screw.

If the nut be fixed and the screw rotated it will advance or come back through a space equal to its pitch on each complete revolution.

If it be required that the screw shall produce pressure, a lever handle of some kind will be necessary in order to turn the screw and to overcome resistance.

Hence we commonly find that the screw and lever are used in combination, and shall take as an example a screw-press such as is used for copying letters.

This apparatus consists of a square-threaded screw, C D, turned by a lever handle, A C B, and working in a nut at E which is part of the frame. Below the screw is an iron slab, which is brought down upon the paper with considerable pressure on advancing the screw through the nut by means of the lever handle. Since the slab is guided by the upright pillars and cannot rotate, the end of the screw terminates in a circular collar, which turns freely in a recess which exactly fits it.



The lever handle is grasped by one hand at B and the other at A, and by a push with one hand and a pull with the other a *couple* comes into action which rotates the screw. This matter has been explained in Art. 53.

100. To find the relation of P to W by the principle of work, let P = pull at B = push at A, and let W be the resistance overcome, then if the screw be single-threaded,

$$\text{P} \times 2\pi\,\text{AC} + \text{P} \times 2\pi\,\text{CB} = \text{W} \times \text{pitch of screw}$$

$$\text{or} \quad \frac{\text{P}}{\text{W}} = \frac{\text{pitch of screw}}{4\pi\,\text{AC}}.$$

101. The relation of P to W in the screw may be deduced from the doctrine of the inclined plane.

We refer to the right-angled triangle, A B C, wound round a cylinder, B C being the pitch of the screw, and suppose a weight W to rest on the portion of thread A B. We shall further suppose that there is a lever of length, a, to which the force P is applied, and that Q is the pressure due to P as felt on the screw thread at a distance b from the axis,

$$\text{then}\quad P \times a = Q \times b.$$

$$\text{Also}\quad Q \times AC = W \times BC.$$

$$\text{Now}\ \frac{Q}{P} = \frac{a}{b} = \frac{\text{circumference of circle described by P}}{\text{circumference of cylinder}},$$

or Q × circumf. of cylinder = P × circumf. of circle described by P.

$$\text{But } Q \times AC = Q \times \text{circumf. of cylinder}$$

$$\therefore\ P \times \text{circumf. of circle described by } P = W \times BC$$

$$= W \times \text{pitch of screw},$$

which is the relation required.

The same result appears at once from the principle of work, for in one revolution,

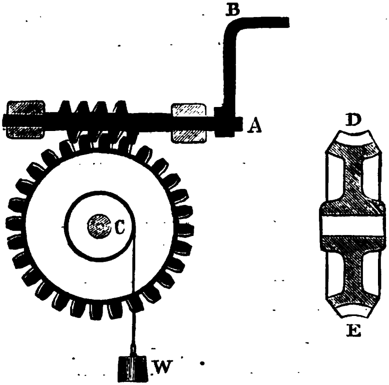
work done by P = P × circumf. of circle described by P
work done on W = W × pitch of screw

∴ P × circumf. of circle described by P = W × pitch of screw.

102. **The endless screw and worm-wheel.**—When a screw is so arranged as to rotate in a fixed frame, it is called an *endless* screw, because the effect of rotation is to cause the threads to appear to travel onwards without any ending.

When used as a mechanical power the endless screw operates with a wheel having oblique teeth cut upon its rim wherewith the thread of the screw engages, and in that case the continuous travel of the so-called worm of the endless screw imparts a constant rotation to the wheel, which is technically known as a *worm-wheel.*

The combination of the endless screw and worm-wheel is shown in the drawing as applied to raising a weight, W, by means of a power, P, applied to the end of the lever handle, A B = b. Let W be attached to a cord fastened to a drum whose centre is C, and radius $= a$, and let n be the number of teeth on the worm-wheel. Also, let the pitch of the screw be so chosen that each turn of the screw advances the wheel by one tooth.



Then for each turn of the lever handle P describes a space $2\pi b$, and the wheel moves through the space of one tooth, whereby W rises through a space, $\frac{2\pi a}{n}$.

$$\text{Hence,} \quad P \times 2\pi b = W \times \frac{2\pi a}{n}$$

$$\therefore \quad \frac{P}{W} = \frac{a}{n b}.$$

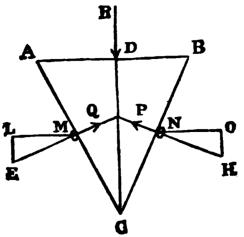
If the screw were a doubled-threaded screw, as is often the case, the wheel would advance through two teeth for each turn of the screw, and we should have

$$\frac{P}{W} = \frac{2a}{n b},$$

whereby the combination is only one-half as effective as before.

103. **The wedge** is a double inclined plane, as ABC in the diagram, which is forced between two obstacles, M and N, tending to come together, and which it is our object to

separate by forcing the wedge into the space between them. We shall consider the wedge to be perfectly smooth.



Let R be the force driving in the wedge, and P and Q the reactions of the obstacles M and N, which balance the thrust R. Then, since there are only three forces in action, they must meet in a point, and for simplicity we shall suppose that P and Q are equal, and that R acts in the vertical line DC, which bisects the angle ACB.

Taking NH, ME to represent P and Q respectively, and drawing NO, ML, horizontal lines, and HO, LE vertical lines, we estimate the action of the wedge as follows:

By the triangle of forces,

$$\frac{P}{R} = \frac{BC}{AB}, \text{ and } \frac{Q}{R} = \frac{AC}{AB}.$$

Let P' be the horizontal thrust at N, and Q' the horizontal thrust at M.

$$\text{Then } \frac{P'}{P} = \frac{ON}{NH} \therefore P' = P \times \frac{ON}{NH} = R \times \frac{BC}{AB} \times \frac{ON}{NH}.$$

$$\text{But } \frac{ON}{NH} = \frac{DC}{BC} \therefore P' = R \times \frac{BC}{AB} \times \frac{DC}{BC} = R \times \frac{DC}{AB}.$$

$$\text{That is, } R = P' \times \frac{AB}{DC}.$$

$$\text{In like manner, } R = Q' \times \frac{AB}{DC}.$$

Cor. This also appears from the principle of work, for let R descend through a space, DC, then

$$\begin{aligned} \text{work done by } R &= R \times DC \\ \text{work done against } P' + Q' &= (P' + Q') \times DB \\ &= 2\,P' \times DB \\ &= P' \times AB. \end{aligned}$$

and these are equal, therefore $R \times DC = P' \times AB$, the same result as before.

CHAPTER VIII.

THE LAWS OF FALLING BODIES—ENERGY—MOTION IN A CIRCLE—THE PENDULUM.

104. **Acceleration.**—Hitherto we have only considered the case of the uniform motion of a body, the variable motion of bodies being beyond our scope. There is, however, one case of variable motion which can be easily dealt with, and that is where a body is continually moving faster and faster according to a simple law, which we proceed to state.

Conceive that the velocity of a body is increased by an equal amount in each successive small interval of time. Thus, suppose that a body is moving at any given instant with a velocity of 10 feet per second, and after $\frac{1}{10}$ of a second has elapsed it is moving with a velocity of 11 feet per second, and after another $\frac{1}{10}$ of a second it is moving with a velocity of 12 feet per second.

Suppose this to continue, whereby at the end of 1 second the velocity would be 20 feet per second, and at the end of 2 seconds it would be 30 feet per second, and so on.

In this way we should obtain the conception of a general uniform rate of increase of velocity; but to test the motion further a like proportionate increase should be observed if we proceeded by dividing a second into 100 or 1,000 equal parts instead of into 10 equal parts, or into any greater number of equal parts.

In such a case the velocity of the body would be said to be *uniformly accelerated*, and the word *acceleration* is employed as follows :—

Definition :—The rate of change of the velocity of a point is called its acceleration.

Thus the student will observe that the rate of change of position is velocity, and the rate of change of velocity is acceleration.

It is evident that acceleration may be uniform or variable. It is *uniform* when the velocity of the point receives equal additions in successive equal periods of time, however small. It is *variable* when these additions are unequal.

The term 'acceleration' is also applied when the velocity of the point is continually retarded; the additions above mentioned are then taken to be negative instead of positive.

Thus, let a body be moving with a velocity 8, and be the subject of a uniform acceleration 3; then

After 1, 2, 3, 4, &c., seconds,
Its velocities will be 11, 14, 17, 20, &c. feet per second
Whereas if it were the subject of a uniform retardation 3
After 1, 2, 3, 4, &c., seconds,
The velocities would be 5, 2, -1, -4, &c. feet per second.

105. The acceleration of a point is *linear* acceleration, and is measured just as we measure linear velocity.

Definition :—The *unit* of acceleration adds a velocity of 1 foot in 1 second.

There are two cases to consider.

(1.) Let the acceleration be uniform, and equal to f.

Since an acceleration 1 adds a velocity 1 in 1 second to the velocity of the point, it appears that an acceleration f will add a velocity f in 1 second.

Therefore, in 2 seconds it will add a velocity $2f$,
and in 3 ,, ,, ,, $3f$;
in t ,, ,, ,, tf.

Let V = velocity of point when acceleration f begins,
v = velocity of point at end of t seconds.

Then $v = \text{v} + ft$.

If the acceleration be negative,

$$v = \text{v} - ft.$$

If $\text{v} = 0$, or the body start from rest, we have

$$v = ft, \text{ or } f = \frac{v}{t};$$

and the amount of acceleration is obtained by the simple division of velocity by time. Thus if a body starts from rest, and has a velocity of 120 feet at the end of 8 seconds, the acceleration being uniform, we have—

$$\text{Acceleration} = \tfrac{120}{8} = 15.$$

(2.) Let the acceleration be variable; it is then measured by the additional velocity (either positive or negative) which would have been imparted in a unit of time if the acceleration had remained throughout that unit the same as it was at the beginning of it.

Further, it is manifest that the term acceleration applies to angular velocity as well as to linear velocity.

106. *Proposition*:—To find the space described by a body moving for a given time with a uniform acceleration.

Let t = time in seconds,
v = velocity before acceleration begins,
f = acceleration,
s = space described in t seconds,
v = velocity at end of t seconds.

To solve this question we employ an artifice.

Let there be two bodies, A and B, starting from the same point and moving in *opposite* directions in the same straight line.

Let A start with velocity v and be subject to acceleration f.
„ B „ „ v „ retardation f.

At end of 1st second A's velocity $= \text{v} + f$
„ 1st „ B's „ $= v - f$
„ t seconds A's „ $= \text{v} + ft$
„ t „ B's „ $= v - ft$

But the velocity with which A and B separate is the sum of these velocities, and is in every second

$$= \text{V} + v, \text{ which is } \textit{invariable.}$$

Hence the distance between A and B after t seconds $= (\text{V} + v)\,t$. But velocity of A increases just as that of B dies away, hence in t seconds the space described by A will equal that described by B.

It follows that distance from A to B in t seconds $= 2$ space described by A in t seconds $= 2\,s$.

But distance from A to B in t seconds $= (\text{V} + v)\,t$

$$\begin{aligned} \therefore\ 2\,s &= (\text{V} + v)\,t \\ &= (\text{V} + \text{V} + ft)\,t \\ &= 2\,\text{V}\,t + ft^2 \\ \therefore\ s &= \text{V}\,t + \tfrac{1}{2}ft^2. \end{aligned}$$

If f be negative. We have

$$s = \text{V}\,t - \tfrac{1}{2}ft^2.$$

Again, to find the relation between v and s. We have

$$\begin{aligned} v^2 &= (\text{V} + ft)^2 \\ &= \text{V}^2 + 2\,\text{V}ft + f^2t^2 \\ &= \text{V}^2 + 2f(\text{V}\,t + \tfrac{1}{2}ft^2) \\ &= \text{V}^2 + 2fs. \end{aligned}$$

If f be negative, $v^2 = \text{V}^2 - 2fs$.

Cor.:—If $\text{V} = 0$, or the body start from rest, we have

$$\begin{aligned} s &= \tfrac{1}{2}ft^2, \\ v^2 &= 2fs. \end{aligned}$$

Hence the formulæ are

(1.) When the body starts from rest,

$$v = ft,\ s = \tfrac{1}{2}ft^2,\ v^2 = 2fs.$$

(2.) When the body has a velocity V before the acceleration begins,

$$v = \text{V} + ft,\ s = \text{V}\,t + \tfrac{1}{2}ft^2,\ v^2 = \text{V}^2 + 2fs;$$

or, $v = \text{V} - ft,\ s = \text{V}\,t - \tfrac{1}{2}ft^2,\ v^2 = \text{V}^2 - 2fs.$

107. **Laws of falling bodies.**—One principal application of these formulæ occurs in the case of bodies falling at the earth's surface, the reason being that the force of gravity produces a constant acceleration in a falling body.

The force of gravity varies a very little at different points of the earth's surface, but for all ordinary purposes it may be considered to be an invariable force, and by the second law of motion the velocity which it generates in any second in a falling body will always be the same. The meaning of this statement is, that if a body falling from rest acquires a certain velocity in *one* second, it will acquire twice that velocity in *two* seconds, three times that velocity in *three* seconds, and so on. But we have agreed to express this fact by saying that the acceleration is constant, and the formulæ which apply where the acceleration is constant have been investigated. It only remains, therefore, for the student to understand how to apply the results of theory to particular cases.

108. When a light body falls in air, the resistance of the air somewhat retards it, but the important law with reference to falling bodies is, that all substances, whether light or heavy, will fall at the same rate in a space from which the air has been exhausted. This has been shown by exhausting the air from a long glass tube, and allowing a sovereign and a feather to fall together within it, when it will be found that they reach the bottom in the same time. But the same thing can be shown without expensive apparatus by making a small paper funnel, loading it with shot, and placing a very light feather on the top of the shot. When the funnel is let fall, the weight of the shot will keep it in an upright position, and the feather and the shot will reach the ground at the same time. Again, three balls of equal size, one of lead, another of wood, and the third of indiarubber, when started together, will strike the ground at the same instant although their weights are very different.

The law just illustrated is a consequence of the fact that gravity is a constant force. If that be so, gravity will impress on each particle of a body the same acceleration, and it matters not whether the number of particles be larger or smaller. The acceleration produced in a feather weighing $\frac{1}{100}$ of a grain is the same as that produced in a pound weight, for if we subdivide the pound it is made up of a sufficient number of hundredths of a grain, and the acceleration of one part is the same as that of any number of parts.

109. The next point is to determine the amount of acceleration produced in a body falling freely in a space from which the air has been exhausted.

This is a matter of experiment, but the result has been ascertained with great accuracy, and at present we may say that, in London, a body falling freely *in vacuo* will describe 16·1 feet in a second of time, or will acquire a velocity of 32·2 feet at the end of the first second of its fall.

This is equivalent to saying that the acceleration produced by gravity is 32·2 feet per second.

In order that questions relating to falling bodies may be distinguished at a glance, it is the custom to appropriate the letter g for denoting the acceleration produced by the force of gravity.

Also since gravity produces a constant acceleration in a falling body, we may at once apply the formulæ proved in Art. 106, merely writing g in the place of f.

Hence when a body falls *in vacuo*, freely and from rest, we have

$$v = gt,\ s = \tfrac{1}{2} gt^2,\ v^2 = 2\,gs, \text{ where } g = 32{\cdot}2.$$

Also when a body is projected vertically upwards with a velocity v, we have

$$v = \text{v} - gt,\ s = t\text{v} - \tfrac{1}{2} gt^2,\ v^2 = \text{v}^2 - 2\,gs.$$

Whereas, when a body is projected vertically downwards with a velocity v, the formulæ become

$$v = \text{v} + gt,\ s = t\text{v} + \tfrac{1}{2} gt^2,\ v^2 = \text{v}^2 + 2\,gs.$$

110. **Examples.**—The following examples will illustrate the formulæ given above.

(1.) Find the velocity acquired by a body falling for $\frac{1}{4}$ second.

Find also the space described, taking $g = 32$. (Science Exam. 1881.)

Here $v = gt = 32 \times \frac{1}{4} = 8$ feet per second.

$$s = \frac{1}{2} gt^2 = \frac{1}{2} \times 32 \times \frac{1}{16} = 1 \text{ foot.}$$

(2.) What velocity will a body acquire in falling freely from rest through 400 feet? ($g = 32$.)

Here $v^2 = 2\,gs = 2 \times 32 \times 400 = 64 \times 400$

$\therefore \quad v = 8 \times 20 = 160$ feet per second.

(3.) The acceleration of a particle is 10, and at a given instant it is moving at the rate of $7\frac{1}{2}$ feet per second.

After what time would it move at 30 feet per second, and what distance would it describe in that time? (Science Exam. 1881.)

Here $v = \text{v} + f$ where $\text{v} = 7\frac{1}{2}, f = 10$

$$\therefore \quad 30 = 7\tfrac{1}{2} + 10\,t$$

or $\quad 10\,t = 22\frac{1}{2}$

$$t = 2\tfrac{1}{4} \text{ or } \tfrac{9}{4} \text{ seconds.}$$

Also $s = t\text{v} + \frac{1}{2} ft^2 = \frac{9}{4} \times 7\frac{1}{2} + \frac{1}{2} \times 10 \times \frac{81}{16}$

or $s = \frac{675}{16} = 42\frac{3}{16}$ feet.

(4.) A body is thrown upwards with a velocity of 110 feet per second. What time willl elapse before it begins to descend? ($g = 32$.)

Here the formula to be taken is $v = \text{v} - gt$.

When the body reaches the highest point, we have $v = 0$,

$$\therefore \quad 0 = 110 - 32\,t, \text{ or } t = \tfrac{55}{16} \text{ seconds.}$$

(4 *continued.*)—After how many seconds will the body be moving downwards with a velocity of 66 feet per second?

Here the formula is $v = gt$.

$$\therefore \quad 66 = 32 \times t$$

$$t = \tfrac{33}{16} \text{ seconds.}$$

Adding the time of ascent to the time of descent, the whole time elapsed in the up and down movement is equal to

$$\tfrac{55}{16} + \tfrac{33}{16} \text{ or } \tfrac{88}{16} \text{ or } 5\tfrac{1}{2} \text{ seconds.}$$

(5.) A train reduces its speed from 40 miles an hour to 10 miles an hour in going 150 yards. The retardation being uniform, what further space will it describe before coming to rest?

Here we must find f from the formula

$v^2 = \mathrm{v}^2 - 2fs$.

$s = 150$ yards $= 450$ feet.

v is 10 miles per hour or 10×5280 ft. in 3600 seconds.

$$\therefore\ v = \frac{10 \times 5280}{3600} = \frac{528}{36} = \frac{44}{3} \text{ feet per second.}$$

v is 40 miles per hour or $\frac{176}{3}$ feet per second.

$$\therefore \text{ by substitution } f = \frac{484}{135}.$$

The train is now moving with a velocity of $\frac{44}{3}$ feet per second, and is subject to a retardation f.

Taking the formula $v^2 = \mathrm{v}^2 - 2fs$, we have $v = 0$ when the train stops. Also $\mathrm{v} = \frac{44}{3}$ feet per second.

$$\therefore\quad 0 = \left(\frac{44}{3}\right)^2 - 2 \times \frac{484}{135} \times s,$$

$$\text{or}\quad s = 30 \text{ ft.} = 10 \text{ yards.}$$

111. **Energy, or accumulated work.**—The work done in raising a weight has been discussed, and we have now to estimate the work done in setting a body in motion.

When force acts upon a free body it will set it in motion, the inertia of the body presenting a resistance which is overcome by the force, and thus work is done in impressing velocity upon a free body. So also a body in motion can only be brought to rest by the action of force, and the point of application of the force moves with the body, whereby work is done during the destruction of the motion.

We reason as follows :—

Let a body of weight w be lifted through a height h, and acquire, in falling down again to the starting level, a velocity v.

Again, let w be thrown vertically upwards with a velocity v, it will rise to the same height h, but no higher, and work will be done in destroying the velocity v.

In both cases wh is the work done, and in both cases

$$h = \frac{v^2}{2g},$$

$$\therefore \quad \mathrm{w}h = \frac{\mathrm{w}v^2}{2g}.$$

We conclude that when a velocity v is impressed on a body of weight w, the work done

$$= \frac{\mathrm{w}v^2}{2g}.$$

112. *Definition*: Energy is the capacity for performing work. Thus a raised weight possesses energy, for we know that we may obtain work from the weight while it is falling. This is seen when we pull up the weight employed to keep a clock going. The weight hangs by a fine cord, which is wound round a barrel; and as the weight descends the barrel slowly turns, and the mechanism of the clock is set in motion. This may go on for several days, the clock keeping time so long as the weight is descending. The energy which exists in a raised weight is called *potential energy*. The word 'potential' is intended to express that a new power has been conferred upon the weight by the act of raising it.

But we have seen that when a raised weight is allowed to fall it acquires a certain velocity, and that there is a definite relation between the height to which the body has been raised and the velocity which it acquires in falling. When we know one, we know the other, the relation being

$$\mathrm{W}h = \frac{\mathrm{W}v^2}{2g}.$$

The expression $\frac{\mathrm{W}v^2}{2g}$ is therefore a measure of the work stored up in a moving body, or of the work which it is capable of doing before its velocity is expended, or before it comes to rest.

The energy, or capacity for doing work, which a moving body possesses is called *kinetic energy*, from a Greek word signifying motion. In fact, kinetic energy is the energy of a body in motion, as distinguished from the energy existing in a raised weight at rest.

Also the student will perceive that energy is only another phrase for work *accumulated*, or stored up in a body, which may be done, either by raising the body through a given height, or impressing upon it a given velocity.

Ex. : Let a velocity of 20 feet per second be impressed on a body weighing 46 lbs. Find the accumulated work as expressed in foot-pounds.

$$\text{Here accumulated work} = \frac{\mathrm{W}v^2}{2g}$$
$$= \frac{46 \times 400}{2 \times 32{\cdot}2}$$
$$= 285 \text{ foot-pounds.}$$

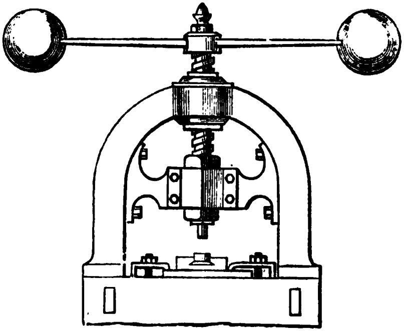
Also, taking the formula $v^2 = 2gh$, we have

$$h = \frac{400}{64{\cdot}4} = 6{\cdot}2,$$
$$\therefore \quad \mathrm{W}h = 285 \text{ foot-pounds ;}$$

the decimal fraction being neglected in both cases.

113. **The fly press** will furnish an illustration of the useful application of the principle of accumulated work. As shown in the diagram, it consists of a strong frame carry-

ing a square threaded screw, which descends vertically upon the work, and is armed with a hardened steel punch die for either punching a hole or impressing some definite form on a yielding piece of metal. The sketch is taken from Mr. Shelley's 'Workshop Appliances,' and shows a hand punch with which two men are able to punch a hole nearly an inch in diameter through the web of a rail perhaps $\frac{3}{4}$ of an inch in thickness.



The lever of the press is a horizontal bar loaded at each end with a heavy ball of metal, which is gradually brought from a state of rest to one of rapid motion by the continued effort on the part of the workmen, the maintenance of its speed being assisted by its descent due to the revolution of the screw.

In examining its action we shall, however, disregard the effect of the descending weight, and confine our view entirely to the work which is accumulated in the revolving balls and expended in driving the punch through the piece of metal.

Let each ball weigh 50 lbs., and have a velocity of 10 feet per second at the instant that the punch begins to operate.

Then accumulated work stored up in the two balls is

$$\frac{2 \times 50 \times 100}{2 \times 32{\cdot}2} = 155{\cdot}3 \text{ foot pounds.}$$

Let the work be expended in driving the punch through $\frac{3}{4}$ of an inch of metal, the resistance R being constant.

Then $\frac{3}{4}$ of an inch $= \frac{3}{4} \times \frac{1}{12}$ ft. $= \frac{1}{16}$ ft.

$$\therefore \ \text{R} \times \tfrac{1}{16} = \text{work done} = 155{\cdot}3$$

$$\therefore \ \text{R} = 155{\cdot}3 \times 16 = 2484{\cdot}8 \text{ pounds.}$$

This shows the great increase of pressure caused by the action of the weights on the lever-arm of the press.

114. **The fly-wheel.**—Just as work is accumulated in the revolving balls of a fly press, so also it may be stored up in the heavy metal rim of a revolving wheel. In such a case the wheel is called a fly-wheel, and is to be met with in machines of every class. Take, for example, such a simple apparatus as a sewing machine driven by hand. The little handle, which is grasped by the operator and gives motion to the mechanism, is fastened on the face of a small iron wheel, having a considerable portion of metal in the rim. When the wheel is turned rapidly, it runs smoothly and steadily, notwithstanding that the force which drives it is continually varying. This exhibits the function of a fly-wheel. Conceive that, for the purposes of rotation, the whole of the metal is collected in one dense circular line at a given distance from the centre,

Let W = weight so collected,
r = radius of the circle,
ω = angular velocity of the wheel.

Then linear velocity of a point in the rim $= \omega\, r$.

$$\therefore \ \text{work stored up in the wheel} = \frac{\text{W}\,\omega^2\, r^2}{2g}.$$

From this equation we can learn something of the nature of a fly-wheel.

(1.) Suppose two fly-wheels exactly alike to be fastened together so as to make one wheel of *double* the weight.

Then the work stored up for a given number of revolutions per minute is doubled.

This shows the influence of weight alone.

(2.) Let the number of revolutions per minute of the wheel be doubled. Then ω becomes $2\,\omega$.

$$\therefore \quad \text{work stored up} = \frac{\text{w} \times 4\omega^2 r^2}{2g},$$

or the work stored up is 4 times as great as before.

If ω became $3\,\omega$, we should have the work stored up 9 times as great as before.

This shows the influence of velocity.

(3.) Let the wheel be enlarged, the weight being the same as before, so that the dense rim may lie in the circumference of a circle of radius $2\,r$.

$$\text{Then work stored up} = \frac{\text{w}\,\omega^2\,4\,r^2}{2g},$$

which again is 4 times as great as before.

Here the law of increase is the same as for the increase of speed, the accumulated work for a given velocity increasing according to the square of the radius.

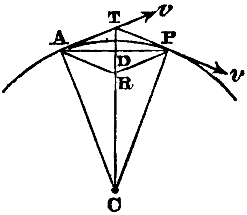
This shows the increased effect due to arrangement of the metal.

We cannot pursue the subject further, but the student who has mastered the formula will hereafter find the study of the advantages to be gained by the use of a fly-wheel to be a simple extension of the elementary facts here laid down, and he will further be interested in ascertaining the exact position of the dense circular line along which the weight may be supposed to be collected.

115. **Motion in a circle.**—The mechanical conditions under which a body moves in a circle will now occupy the student, and the most familiar instance that he will meet with will probably be when he whirls a stone round in a sling. In such a case the string is stretched tight, and he feels a constant pull upon the hand. But here the problem is, in fact, complicated by the force of gravity consequent on the weight of the stone, and in order to arrive at the theory we must take an ideal case, such as that of a body describing in some imaginary way a circle upon a smooth horizontal plane.

Proposition : A body of weight w, describes a circle of radius r, with a uniform velocity v, to find the direction and magnitude of the force which produces this motion.

It is clear that some force must act, or, by the second law, the body would describe a straight line and not a circle.



Let A P be an arc of the circle described round C in the very small time t. Draw A T, P T tangents at A and P, complete the parallelogram A T P R, and draw the diagonals T R, A P intersecting in D. By a property of the circle, we have A T = T P, whence A T, T P may represent in magnitude and direction the velocity v, of the body at the points A and P respectively.

It follows that the force must have impressed on the body a velocity T R during its motion from A to P. But T R, when produced, passes through C, hence the force must reside in the centre of the circle.

Let F represent its magnitude.

Then F generates a velocity T R in time t.

But W „ „ $g t$ „ t.

$$\therefore \quad \frac{F}{W} = \frac{TR}{g t}.$$

Again, by a property of the circle, we have triangle T R A similar to triangle P C A.

$$\therefore \quad \frac{TR}{AT} = \frac{AP}{CA} \text{ or } \frac{TR}{v} = \frac{AP}{r}.$$

But since the time t is supposed to be the smallest possible, A P is a very small arc, and we may, without error, assume that arc A P = chord A P, also arc A P = $t v$.

Whence $$\frac{TR}{v} = \frac{t v}{r} \text{ or } TR = \frac{t v^2}{r}.$$

But $$F = \frac{W}{g t} \times TR$$

$$\therefore \quad F = \frac{W}{g t} \cdot \frac{t v^2}{r} = \frac{W v^2}{g r}.$$

Ex. : A weight of 12 lbs. is fastened to one end of a string 4 feet long, the other end of the string being attached to a fixed point C in a smooth horizontal plane. The body moves with a velocity of 10 feet per second, find the tension of the string.

Here, tension of string $= \frac{12 \times 100}{32{\cdot}2 \times 4} = \frac{300}{32{\cdot}2} = 9{\cdot}3$ lbs.

It appears from the proposition just investigated, and is also made clear by the example, that when a body fastened to a string moves uniformly in a horizontal circle the tension of the string tending to pull the body in towards the centre will be

$$\frac{\text{W}\,v^2}{g\,r}.$$

But the string is attached to a fixed point C, which supports its tension. It follows that while the body experiences a constant pull inwards, the centre C is at the same time subject to an equal and opposite pull outwards.

The outward pull on the centre is called a *centrifugal* or centre-flying force, and is equal to

$$\frac{\text{W}\,v^2}{g\,r}.$$

Much confusion has been introduced into this subject by treating the centrifugal force as something acting upon the body. No such thing happens. The body is always tending to go on in a straight line, and is kept in a circle by a force continually pulling it in towards the centre.

The centrifugal force, or tendency to move in the direction of the radius, is felt only by the support to which the body is attached. There are several ways of showing this pull on the centre and the effect which it produces.

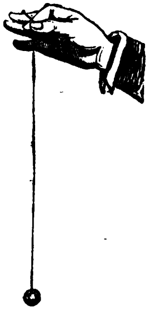
Take a cylinder of wood about 5 inches in diameter and 3 inches in length, excavate a hole about an inch deep on the rim, and fill it up with lead, retaining the circular form of the rim. Then set the cylinder to roll on a horizontal plane, and the pull on the centre will be evident enough.

Again, prepare a disc of wood about 8 inches in diameter, thread it on a wooden bar about an inch in diameter, and run a stout iron wire along the axis or central line of the bar. The ends of this wire may rest in two iron rings supported on pillars.

The disc will, if well shaped, rotate smoothly and steadily in its bearings when set spinning by a string.

But now load the rim with a small lump of lead or brass, and put the disc into rapid rotation. It will be found to jump in its bearings under this pull on the centre acting always in a line pointing towards the revolving weight, and the disturbance is so unexpectedly complete that it becomes almost ludicrous.

116. **The simple pendulum.**—An imaginary or *simple pendulum* is a particle of matter suspended by a string without weight. Such a pendulum can only exist in the imagination, but we can roughly determine its properties by suspending a small ball by a fine string, as in the sketch. If this be done, it will at once appear that the time which the ball occupies in making a complete swing from right to left depends upon the length of the string, being greater as the string is lengthened.



The distance from the point of suspension to the centre of the ball is called the *length* of the simple pendulum.

The *oscillation* of a pendulum is the whole movement which it makes in one direction before it begins to return.

The *time of an oscillation* is the period of this movement. Where a body swings in a circle, as in the case supposed, the time of an oscillation is to a small extent affected by the length of the arc through which it swings, and mathematicians have shown that there is one particular curve, known as a *cycloid*, in which the swing of the pendulum would be the same for all arcs, whether great or small.

Investigations of this kind are not suitable for the present treatise, and we must be content merely to state the *pendulum law* as it obtains when a body swings in a cycloid, or through a small arc in a circle, which is practically the same thing.

Let l be the length of a simple pendulum,
t the time of an oscillation,

Then $$t=\pi\sqrt{\frac{l}{g}}$$

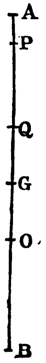
It follows that if there be two pendulums of lengths 1 and 4, their times of oscillation will be as 1, 2.

Let L be the length of a pendulum oscillating seconds, technically known as a seconds pendulum,

Then $$1=\pi\sqrt{\frac{L}{g}}$$

The value assigned to L is 39·14 inches in London.

117. We have next to consider the oscillation of a heavy rod AB suspended at one end A. We shall suppose the rod to be uniform and rigid, and remark that it is made up of a number of particles such as P, Q, each at different distances from the point A, and therefore all tending to swing in different times. But the rod being rigid, every particle must swing in the same time; a compromise will therefore be effected, and the rod will oscillate as if its whole mass were collected into one dense particle at some point O, intermediate between A and B.



The rod AB is termed a compound pendulum, because it is made up of the aggregate of a number of simple pendulums.

The position of O is found by a formula:—

Let G be the centre of gravity of AB, and let AG $=h$.

Also let k^2 be the sum of the squares of the distances of each particle of A B from the point G.

Then $$\text{A O} = \frac{k^2 + h^2}{h}.$$

118. A very remarkable property of the point O was discovered by Huyghens, namely, that the rod A B will swing in the same time whether hung from the point A, or turned upside down and suspended from the point O.

The point O is called the *centre of oscillation*, and the point A is the *centre of suspension*; hence the discovery made by Huyghens is expressed by saying that the centres of suspension and oscillation are convertible. That is, if A be the centre of suspension, O will be the centre of oscillation; or if O be the centre of suspension, A will be the centre of oscillation; and in each case

A O = length of equivalent simple pendulum.

The quantity k is found by analysis, and k is a line of definite length dependent on the form of the suspended body. It is technically known as the *radius of gyration*.

In the case of a uniform rod, $k = \frac{1}{\sqrt{12}}$ (length of rod).

Let A B be a uniform rod, and let A B $= a$.

Then $k^2 = \frac{a^2}{12}$, also $h = \frac{a}{2}$, or $h^2 = \frac{a^2}{4}$;

$$\therefore \quad \text{A O} = \frac{\frac{a^2}{12} + \frac{a^2}{4}}{\frac{a}{2}} = \frac{\frac{a^2}{3}}{\frac{a}{2}} = \frac{2a}{3}.$$

119. An approximate value of g may be readily found by the aid of a pendulum. In a rough experiment, made to supply a result for the present paragraph, a small clip was screwed into the frame of a doorway, and from it was hung by a silk thread a small leaden ball 1½ inch in diameter. It was estimated that the distance from the point of suspension to the centre of the leaden ball was 83 inches.

The arc of swing of the pendulum was about 8 inches, and it made 103 double or 206 single vibrations in 5 minutes, that is, in 300 seconds.

$$\text{Then time of vibration} = \pi\sqrt{\frac{83}{12g}} = \frac{300}{206} \text{ also.}$$

$$\text{Hence} \qquad \frac{\pi^2 \times 83}{12g} = \frac{150 \times 150}{103 \times 103},$$

$$\text{or} \qquad g = \frac{8690646{\cdot}6712}{270000} = 32{\cdot}188.$$

It is estimated that the correct value of g in London, or more accurately at Greenwich, is 32·1912.

120. **Kater's pendulum experiment** is one of great historical interest, as it was the first elaborate attempt to verify the law of Huyghens, and obtain from it a measurement of the length of a seconds' pendulum, and by inference the exact numerical value of g. Captain Kater's experiments were finished in London in the year 1818, and were simple in character. A bar of brass was loaded with a bob and with two small adjustable weights, the centres of suspension and oscillation of the bar with its weights being marked by triangular wedges of steel, technically called knife-edges. The pendulum, being placed with a knife-edge resting on the plane supports, was caused to swing through a small angle in company with another pendulum driven by a clock, and the coincidences of the two pendulums were observed by a telescope.

The object was to get the centre of oscillation exactly at the point of the knife-edge, which was done by shifting the sliding weight until the time of swing became the same from whichever knife-edge the pendulum was suspended. Then the distance between the knife-edges gave the length of the pendulum, and the time of a vibration was known by comparison with the clock. It followed that the length of a pendulum oscillating in a given time was known, and that the length of a second's pendulum could therefore be determined.

The value of L given by Kater was 39·1393 inches. Also taking the formula $1 = \pi \sqrt{\frac{L}{g}}$ and remembering that g in feet is represented by $12g$ inches, we have

$$1 = \pi \sqrt{\frac{39{\cdot}1393}{12g}}$$

$$\text{or } g = \frac{\pi^2 \times 39{\cdot}1393}{12}$$

Now $\pi = 3{\cdot}14159$; and $\pi^2 = 9{\cdot}8698$

$$\therefore g = \frac{9{\cdot}8696 \times 39{\cdot}1393}{12} = 32{\cdot}1908.$$

The exact value of g at Greenwich is taken to be 32·1912, and it is to be noted that the length of a seconds' pendulum varies in different latitudes; thus it is 39·1285 inches in Paris, 39·1654 inches in Stockholm, 39·1043 at Barcelona, the respective values of g being 32·1819, 32·2122, and 32·1620.

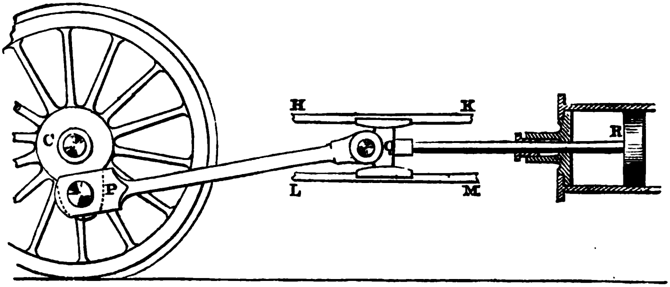
The general result of observation shows that g diminishes as we travel southwards, (say) from Lapland in latitude 70° to the equator. This variation is due to two causes—(1) the earth rotates on its axis once in twenty-four hours; (2) the earth is not an exact sphere, but is flattened at the poles. These causes are shown by astronomers to account for the observed variation in the force of gravity, which is, however, small, for Sir John Herschel states that a body weighing 194 lbs. at the equator will weigh 195 lbs. at the pole. That is, the difference between the equatorial and polar weights of one and the same mass of matter is 1 part in 194 of its whole weight.

CHAPTER IX.

ELEMENTARY MECHANISM.

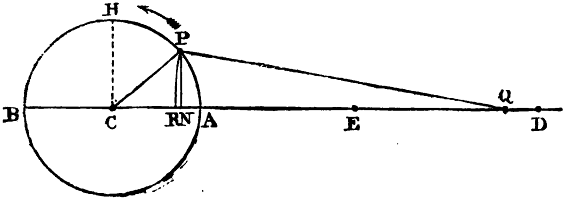
THE problem of the conversion of circular into reciprocating motion has already been placed before the student in the most simple form which it can assume, and it will be convenient now to classify and arrange certain elementary pieces of mechanism or mechanical contrivances for the conversion of motion. We begin with a device commonly employed in the steam-engine, where it may serve either to convert circular into reciprocating motion or to change reciprocating into circular motion.

121. **The crank and connecting-rod** are shown in the diagram as applied in a locomotive engine. The crank, C P, is part of the driving-wheel, and the connecting-rod, P Q, is attached by a pin to the crank, C P, at P, and to a sliding piece at Q, which is constrained to move in a horizontal line by the guides H K, L M.



Here the pressure of steam on the piston in the steam cylinder causes Q to oscillate between the guides, and thereby rotates the driving-wheel.

It often happens that the rotation of CP is employed to cause a to-and-fro movement in Q, and our first object will be to point out that in such a case the motion of Q is affected by the varying inclination of PQ.



Let CP be the crank, PQ the connecting-rod, and let APB be the circle described by P. As before, the point Q is constrained to move in the line DB, passing through C.

Draw PN perpendicular to AB, and let D and E be the extreme positions of Q, then DE = AB.

Also AN is the resolved part of the motion of P in AP.
,, RN ,, ,, ,, P in RP.

$$\therefore \quad DQ = AN + RN = AR.$$

Hence Q has the motion of R instead of that of N.

If PQ remained parallel to BD throughout the motion, the point Q would have precisely the same movement as N.

But such a thing could only happen if PQ were infinite in length; hence the motion of N is often spoken of as being that of a crank with an *infinite link*, the word 'link' being used to express the connecting-rod.

It is of course inconvenient to have a connecting-rod of any great length, but there are contrivances in practice for obtaining the so-called motion of a crank with an infinite link, and one of them has already been put before the reader in Art. 67.

In order to show the effect produced by the angular position of P Q.

With centre Q and radius Q P describe a circular arc, P R cutting A B in R.

Draw C H at right angles to A C B, and let E be the position of Q when P arrives at the point B. Then D E is the whole space described by Q, and is called the *stroke* of the point Q.

It follows that when C P comes to the position C H, or the crank has made one-fourth of a revolution, the point Q is a little in advance of the middle of its stroke.

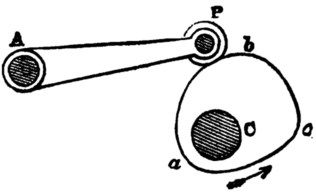
This fact exercises an important influence on the working of a steam-engine.

In the steam-engine, Q is connected directly with the piston of the steam cylinder, and it is evident that, when Q is at either of the points D and E, the pressure on the piston, which acts in the line D A C B, will not be able to move C P, for it will be a direct thrust or else a direct pull along the crank itself, and has no tendency to turn the crank. Hence the points A and B are technically known as *dead points*, being points where the driving force ceases to operate.

122. **Cams.**—Circular may be converted into reciprocating motion by the aid of cams.

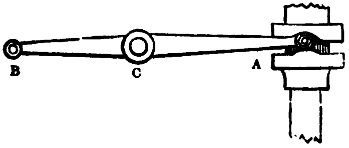
The term 'cam' is applied to a curved plate or groove which communicates motion to another piece by the action of its curved edge.

Such a plate is shown in the sketch, and is marked *a b c*, the centre of motion being at C. It will be understood that the cam is mounted on a shaft running through C, and that its object is to give an up-and-down movement to the lever A P, whose fulcrum is at A. There is a roller or bowl, P, on the end of the lever in order to diminish the friction on the cam-plate.



In the drawing, the portions *a b*, *c a*, are any given curves, and *b c* is a portion of a circle described about C as a centre.

It is easy to understand that, during the rotation of the cam in the direction of the arrow, the roller, P, at the end of the lever, A P, will be gradually raised by the curved portion *a b*, will be held at rest while *b c* passes underneath it, and, finally, will be allowed to fall by the action of *c a*. In this way a cam may be made to impart any required motion, and may reproduce in machinery those delicate and rapid movements which would otherwise demand the highest effort of skill from a practised workman.

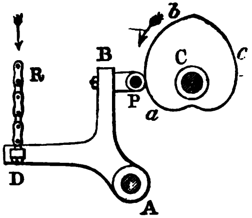


The cam, referred to above, communicates motion in a plane perpendicular to the axis about which it rotates, but it often happens that the motion is to be imparted in a direction parallel to the driving axis. In such a case the cam will no longer take the form of a plate, but will be a groove running round a circular cylinder. Such a cam is shown in the sketch, where A C B is a lever having a fulcrum at C, and movable in the plane of the paper. The end B communicates with some mechanism, and the end A is driven by the cam, which takes the form of a groove running round the cylinder whose axis is parallel to the plane of the paper. It will be understood that, so long as this groove does not deviate from a circular ring whose plane is perpendicular to the axis of the cam-shaft, the lever will remain at rest, but the moment the groove departs in the least degree from this exact form, as indicated in the drawing, an up-and-down movement of the lever arm must take place. The deflection in the drawing indicates a rapid up-and-down movement of the lever arm C A.

123. **The heart-wheel** is a form of cam which has been much used in spinning machinery, and serves to convert the uniform rotation of a shaft into an up-and-down move-

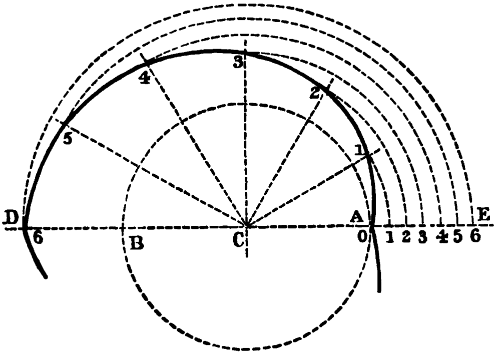
ment, which is also *uniform*. One half of the cam gives the upward movement and the other half gives the downward movement.

The drawing shows a heart-wheel, centred at C, and rotating in the direction of the arrow. The roller, P, relieves the friction between the cam-plate and the piece on which it acts. A forked lever, B A D, centred at A, is pushed to the left by the portion *a b* of the cam, whereby the chain, D R, is pulled down. As the part *b c* comes under P, the forked lever returns at a rate determined by the shape of the cam, the necessary contact between P and the plate being kept up by the action of a weight or spring.



Inasmuch as the form of the cam ought to be set out with accuracy, we shall show the geometrical construction for determining it.

For simplicity, suppose that the roller, P, has diminished to a point, and that it is required to move P uniformly from A to E and back again as the cam rotates.



Let C be the centre of motion, describe a circle with radius C A, and set off the six equal angles, A C 1, 1 C 2, and so on, each of which is equal to 30°.

Divide A E into a like number of equal parts, marked 1, 2, 3, &c., and with centre C and radius C 1, measured on the line C E, describe a circular arc meeting the inclined line C 1, and construct in a like manner for each of the points 1, 2, 3, &c., on the line C E. The curve of the cam will pass through the points 1, 2, 3, as shown by the full line in the diagram.

We have set out six points only, but of course the number may be increased if greater accuracy be required.

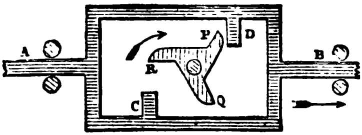
In order to allow for the use of a roller, we should, after setting out the curve as above, describe a number of small circles, each of which represents a section of the roller, and whose centres lie in the curve set out. An inner curved line resembling the original cam, but somewhat smaller, and touching the inside portion of each circle, will give the actual cam-plate required.

124. **Escapements.**—Circular may be converted into reciprocating motion by the aid of escapements.

An *escapement* consists of a wheel fitted with teeth which are made to act upon two distinct pieces or pallets attached to a reciprocating frame, and it is arranged that when one tooth escapes or ceases to drive a pallet, the other shall commence its action.

One of the most simple forms is the following:—

A sliding frame, A B, is furnished with two projecting pieces at C and D, and within it is centred a wheel having three teeth, namely, P, Q, and R. This wheel tends always to turn in the direction indicated by the arrow, and taking the position shown in the diagram, it is apparent that P is in the act of pressing D to the right. As the wheel continues to turn, P will slip off



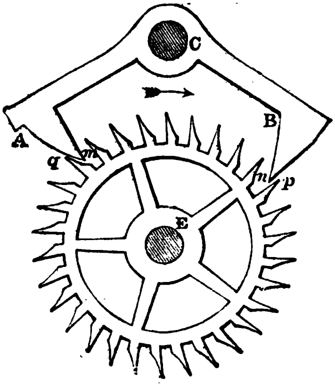
D or *escape*, and Q will strike against C, and will drive A B to the left hand. Presently Q escapes, and R comes in contact with D, whereby the frame is again driven to the right. Thus the rotation of the wheel causes a reciprocating movement in the sliding piece A B.

It is clear that the wheel must have 1, 3, 5, or some odd number of teeth upon its circumference.

125. **Anchor or recoil escapement.**—A great deal might be written upon escapements if there were more space at our disposal, but at present we only give one example, being the form of escapement invented by Dr. Hooke, the contemporary of Newton, and first applied when the pendulum came into use as a regulator of clocks. This device is called an anchor escapement, because the reciprocating frame, A B, mentioned in the last article, takes the form of an *anchor*, and sometimes it is called a *recoil* escapement, because there is a defect in its action producing a recoil.

Looking at the diagram of Hooke's invention—

(1.) There is a wheel with pointed teeth, called an escape-wheel, centred at E, and tending to turn in the direction of the arrow.



(2.) There is an anchor, A C B, centred at C, and provided with projections or pallets, A *m*, B *n*, which engage with the teeth of the wheel.

These pallets may be flat or slightly convex, but they are subject to the condition that the perpendicular to A *m* shall pass above C, and the perpendicular to B *n* shall pass between C and E. The point of a tooth is represented as having escaped from the pallet B *n* after driving the anchor to the right hand, and the point *q*, by pressing against A *m*, is supposed to have already pushed the anchor a little to the left hand, and thus the circular motion of the wheel sets up a vibratory or to-and-fro movement of the anchor upon the centre, C.

If the escape-wheel engaged only a light metal anchor, such as that shown in the drawing, the rapidity of the vibration set up would be very great, but in a clock the object is to provide that the wheel shall advance by measured and regular steps, and the anchor is therefore controlled by the inertia of a comparatively heavy pendulum, which swings with it.

There is one uniform method of connecting the anchor and pendulum which may be seen in any clock. The pendulum, consisting often of a compound metal rod with a heavy bob, is hung on a piece of flat steel spring, and swings in a plane behind the anchor from a different point of suspension. To the anchor is attached a light vertical rod with a forked end bent so as to embrace the pendulum rod. The result is that the anchor and pendulum swing together as one piece, although each has a separate point of suspension.

The function of the pendulum will now be understood. As the pendulum swings to and fro it receives an impulse from the pointed teeth of the escape-wheel, first in one direction and then in the other, each impulse being given while the point of a tooth is sliding down the surface B *n* or along the surface A *m* in the direction from A to *m*. This impulse maintains the swing of the pendulum and prevents it from dying away.

But the pendulum acts as a check on the rotation of the scape-wheel. As soon as a tooth falls (say) on B *n*, it can go no farther than B *n* allows it to move; and here it is that a recoil is set up. When the tooth first strikes B *n*, the pendulum is swinging to the left, but the swing is not quite finished, whereby the inertia of the pendulum carries the tooth back a little, and the scape-wheel recoils until the pendulum reaches the end of its swing. At that instant the tooth presses the pendulum onwards and an impulse takes place. The like happens on the pallet A *m*.

This recoil is injurious for reasons which are beyond our scope, and it has been avoided by a different and improved construction of the pallets of the anchor, the result being that the time kept by a clock is sufficiently exact for the purposes of astronomers, who require to measure time with the utmost precision. The technical name for the improvement is the 'dead-beat escapement.'

The recoil escapement, in some shape or other, is applied separately and without any pendulum for the alarum of a clock. A hammer which strikes a bell is connected with the anchor, and is set into rapid vibration as the scape-wheel runs down. In this way blows are struck in quick succession by the hammer upon the bell, and the sound of the alarum is set up.

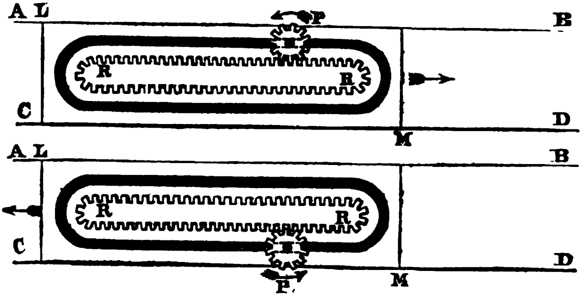
Finally, we remark that the recoil escapement may be used also to convert reciprocating into circular motion, for if the anchor be moved to and fro it will be found that the scape-wheel turns round always in one direction.

126. **Mangle motion.**—Mangle-wheels or racks form a separate class of contrivances for the conversion of circular into reciprocating motion.

There is only space for one example, which is shown in the diagram, as taken from a model belonging to the Normal School.

The lines A B, C D represent two horizontal guide-bars in which the block, M, moves by a sliding motion. P is a pinion turned by a winch-handle, and rotating always in one direction, as shown by the arrow. This pinion is swung on a bar having a fixed centre, so that it can be shifted from the position shown in the upper part of the diagram to that shown in the lower portion, and its axis is guided by a pin running in the shaded groove in such a manner that its teeth always engage with the double rack, R R, having rounded ends.

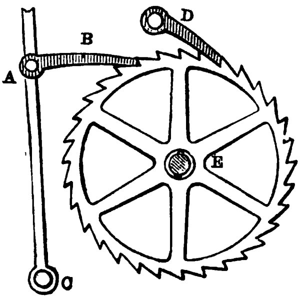
In the upper sketch the block is moving to the right under the action of the pinion P, whereas in the lower sketch the pinion has run round the curved end of R R, and is engaging with the lower portion of the rack, whereby the sliding block is moved to the left, that is, in the opposite direction. Hence the continuous circular motion of the pinion, P, communicates an endlong to-and-fro movement to the sliding block.



We pass on to a class of contrivances which are employed in converting reciprocating into circular motion, being the converse problem to that last considered.

127. **A ratchet-wheel,** as usually made with saw-shaped teeth, which is driven by a click or paul, B, and is prevented from recoiling or running back by a detent, D. The driver, B, may be mounted on a vibrating arm, C A, which moves to and fro through a small angle on a centre of motion, C.

A ratchet-wheel, when made as shown in the diagram, can only be driven in one direction. As A C moves to the right hand, the paul, B, pushes round the wheel through a certain angle. Upon the return of A C, the paul, B, slides over the points of the teeth, and is ready again to push the wheel round through the same angle as before, being in all cases pressed against its teeth by its weight or by a spring.



The detent, D, prevents the wheel from going back while B is moving over the teeth, and is employed when the wheel is liable to recoil, but in many cases, as where the friction of the parts with which the ratchet is connected is sufficient to hold it, the detent may be dispensed with.

In this contrivance, circular motion is derived from reciprocating motion by the simple expedient of suppressing one half of the reciprocating movement and allowing only the other half to be operative.

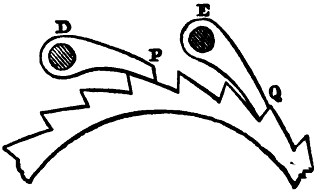
The wheel, E, and the vibrating arm, A C, are often centred upon the same axis.

A common method of applying a ratchet wheel in ordinary mechanism is to be found in the drawing of a winch or crab, as given in a subsequent article. The object of the ratchet is to prevent the recoil of the barrel when the handle is let go.

There is a similar application of the ratchet-wheel to capstans, where the power is exerted on a number of bars or handspikes, standing out like the spokes of a wheel.

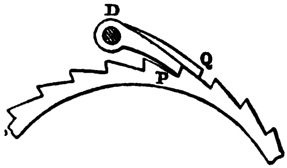
In the case of a capstan, where the resistance overcome is often very great, it was an early improvement to provide two pauls or detents of different lengths, termed by the sailors *paul and half paul*, and thereby to hold up the barrel at shorter intervals, during the winding on of the rope; in fact, a ratchet-wheel of 8 teeth thus became practically equivalent to one of 16 teeth, and the men were better protected from any injury which might be caused by the recoil of the handspikes.

The principle of this contrivance is very intelligible, and is shown in the sketch, where the two pauls, D P, E Q, differ in length by half the space of a tooth.



As the wheel advances by intervals of half a tooth, each paul falls alternately, and the same effect is produced as if the number of teeth were doubled, and there was one paul.

In like manner three pauls might be used, each differing in length by $\frac{1}{3}$ the space of a tooth, and so the subdivision might be extended.

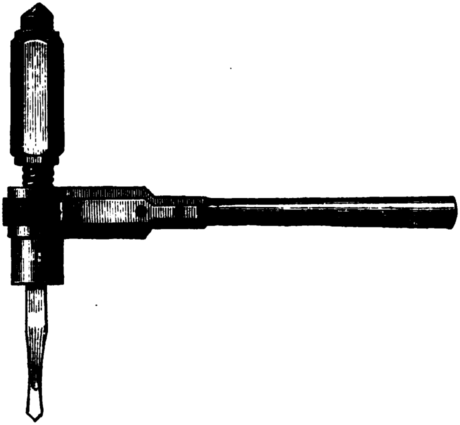


The same principle may be applied to the driving paul, and in this way a ratchet with strong deep teeth, say 30 in number, may be made equivalent to a wheel with 60 teeth. For this purpose the clicks, D P, D Q, are hung upon the same pin, D, and differ in length by half the space of a tooth. It is manifest that they will engage the teeth alternately, and that the wheel may be advanced by half the space of a tooth at each stroke, or, if preferred, by $1\frac{1}{2}$, $2\frac{1}{2}$ teeth at each stroke, and so on.

And the similar construction may be applied in the case of 3 clicks, whereby the ratchet wheel is virtually a wheel having three times the original number of teeth.

128. An example of the use of a ratchet is to be found in the *ratchet-brace*, which is a portable instrument for drilling holes in metal.

It is represented in the diagram, taken from Mr. Shelley's 'Workshop Appliances,' and the ratchet is placed on the outside of the stock which carries the drill. The first thing to be done is to fix the work, or piece of metal operated on, so that it cannot move, and the instrument itself is held in position by a *cramp*, or bent bar of iron, firmly fixed in such a manner that he point of the drill rests on the work, and the upper end of the brace, which terminates in a pointed cone of hard steel, abuts against an arm of the cramp. The drawing shows the drill, fixed into a stock, on the upper part of which is the ratchet employed for rotating the drill. There is a spring paul or detent pressing against the teeth and fixed to the handle, whereby the ratchet is driven when the handle is moved forward, and the paul slides over the teeth as the handle is carried back. In this way a to-and-fro movement of the handle produces an intermittent rotation of the drill spindle.

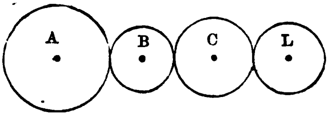


The remainder of the instrument consists of a contrivance for advancing or feeding on the cutter as the drill enters the work. For this purpose a screw comes into play, and it will be seen that the upper part of the drill stock terminates in a screw which enters a long nut with parallel sides. When this nut is not grasped by the hand, but is left free, the whole instrument, consisting of the stock, screw, and nut, rotates as one piece. But as soon as the workman grasps the nut or holds it with a key and prevents it from rotating, the screw is brought out and the point of the drill is pressed more strongly into the work. Thus the rotation of the drill is provided for by the ratchet, and the advance of the cutter is due to the action of a screw. It is from simple combinations of this kind that machine tools, which are so valuable in practice, are constructed.

129. **Wheels in trains.**—When a series of wheels are in gear, we have a *train of wheels.*

It has been shown that a wheel and pinion upon the same axis is a combination equivalent to a lever with unequal arms, and modifies the force which may be transmitted through it; whereas a single wheel is equivalent to a lever with equal arms, and produces no modification in the force which may pass through it.

If, therefore, we have a train of wheels, A, B, C, L, as in the sketch, no two wheels being on the same axis, the train is equivalent to a single pair of wheels, viz. the first A and the last L. The intermediate wheels act as carriers only, and produce the same effect as if A and L were connected by a band.



Also since two wheels in gear turn in opposite directions, it follows that C turns in the same direction as A, but that B and A turn in the opposite direction; that is to say, each additional wheel inserted between A and L changes the direction of the rotation of L.

In a train of wheels, let A be the first driver and L the last follower, and let L make e revolutions while A makes one revolution, then

$$e = \frac{\text{number of revolutions of L in a given time}}{\text{number of revolutions of A in same time}}.$$

The ratio e, which may be a whole number or a fraction, is called the *value of the train,* and it can be determined when the arrangement of the train and the numbers of teeth upon the respective wheels are given.

Let there be two wheels, A and B, in gear; let A have m teeth and B have n teeth. Then

$$\frac{m}{n} = \frac{\text{circumference of pitch circle of A}}{\text{circumference of pitch circle of B}}.$$

Therefore, $\dfrac{\text{number of revol. of B in a given time}}{\text{number of revol. of A in same time}} = \dfrac{m}{n},$

or, in this case, $e = \dfrac{m}{n}$.

Next, let A, B, C, L be in gear, as in last diagram, each wheel being on a separate axis, and let A, B, C, L have m, n, r, l teeth respectively.

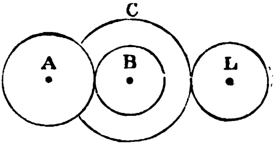
Then $\dfrac{\text{no. of revol. of B}}{\text{no. of revol. of A}} = \dfrac{m}{n}$, $\dfrac{\text{no. of revol. of C}}{\text{no. of revol. of B}} = \dfrac{n}{r}$, $\dfrac{\text{no. of revol. of L}}{\text{no. of revol. of C}} = \dfrac{r}{l}$. Whence, by multiplication,

$$\frac{\text{no. of revol. of L}}{\text{no. of revol. of A}} = \frac{m}{n} \times \frac{n}{r} \times \frac{r}{l} = \frac{m}{l},$$

$$\text{or} \quad e = \frac{m}{l},$$

and the value of the train is the same as if A and L were the only two wheels in gear.

Next, alter the disposition of the wheels and put B and C on the same axis, as in the sketch. Then A and B together make a driver and follower, and so do C and L. Also B and C rotate together as one wheel.



Hence $\dfrac{\text{no. of revol. of B}}{\text{no. of revol. of A}} = \dfrac{m}{n}$, $\dfrac{\text{no. of revol. of L}}{\text{no. of revol. of C}} = \dfrac{r}{l}$,

and $\dfrac{\text{no. of revol. of B}}{\text{no. of revol. of A}} \times \dfrac{\text{no. of revol. of L}}{\text{no. of revol. of C}} = \dfrac{m}{n} \times \dfrac{r}{l}$,

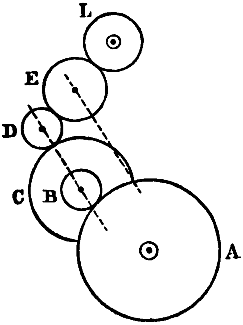
$$\therefore \quad \frac{\text{number of revolutions of L}}{\text{number of revolutions of A}} = \frac{m}{n} \times \frac{r}{l},$$

$$\text{or} \quad e = \frac{m}{n} \times \frac{r}{l}.$$

And the same would be true for any number of wheels; hence we have the following rule, which holds good when a number of wheels are in gear, viz.:

$$e = \frac{\text{product of numbers of teeth on drivers}}{\text{product of numbers of teeth on followers}}.$$

Ex.: Take the following train from Mr. Shelley's 'Workshop Appliances,' as employed in screw-cutting:—



Let A have 80 teeth
„ B „ 20 „ } B and C being on the same axis
„ C „ 60 „
„ D „ 21 „ } D, E, and L being on separate axes.
„ E „ 35 „
„ L „ 30 „

Then A, C, D, E are all drivers
B, D, E, L are all followers.

$$\therefore \; e = \frac{80 \times 60 \times 21 \times 35}{20 \times 21 \times 35 \times 30} = 8.$$

Or L makes 8 revolutions while A makes 1 revolution.

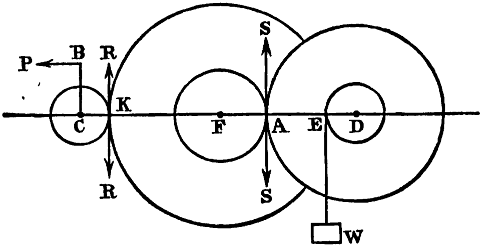
As to the direction of rotation, where a train of wheels are in gear, the 1st, 3rd, 5th, &c. turn in the same direction, while the 2nd, 4th, &c. turn in the opposite direction.

In the example just given, L is the 5th wheel, and therefore turns in the same direction as A.

130. The investigation here concluded will make clear the use of a train of wheels as modifying motion, but it is further necessary to consider the effect of a train of wheels as modifying the transmission of force, and for this purpose it may suffice to find the relation of P to W in the combination sketched, where a pair of toothed wheels, K and F, having

a common axis at F, is interposed between the wheels C and A. As before, the wheels are represented by their pitch circles, and let R be the action or reaction at K, and S the same at A. Then

$$P \times CB = R \times CK$$
$$R \times KF = S \times FA$$
$$S \times AD = W \times ED.$$



$$\therefore \quad PRS \times CB \times KF \times AD = RSW \times CK \times FA \times ED$$
$$\therefore \quad P \times CB \times KF \times AD = W \times CK \times FA \times ED,$$
$$\text{or} \quad \frac{P}{W} = \frac{CK \times FA \times ED}{CB \times KF \times AD}.$$

Let m n be the numbers of teeth on C, F, respectively
,, s, t ,, ,, ,, K, A, ,,

$$\text{Then} \quad \frac{P}{W} = \frac{m \times n}{s \times t} \times \frac{ED}{BC}.$$

It will be seen that the wheels C and F are the drivers, and K and A are the followers. Also that we might extend this combination by inserting another pair of unequal wheels between F and A, taking care that both were placed on the same axis.

Let r, v be the numbers of teeth on the new driver and follower respectively, then we should have

$$\frac{P}{W} = \frac{m \times n \times r}{s \times t \times v} \times \frac{ED}{BC}.$$

Whence the gain of power by wheel work becomes apparent.

[illegible] e lever in [illegible] barrel or

[illegible] onging to [illegible] iron stan- [illegible] heelwork, [illegible] pinion, C, [illegible] eth. On [illegible] d which a [illegible] ound up. [illegible] s squared [illegible] of which, [illegible] lever, A B, [illegible] rum is 4½

[illegible] e exerted [illegible] have

[illegible]

[illegible] machine;

[illegible]

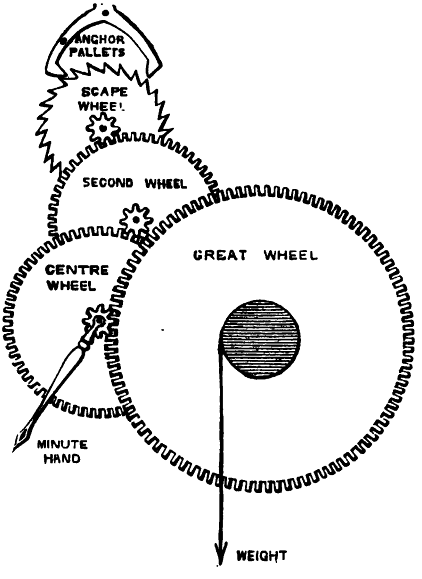
and it only remains for us to consider the adjuncts, which are :—

1. A ratchet-wheel and paul, P, placed at one end of the drum, whereof only the paul, P, comes out in the drawing ; the object of the ratchet and detent being to prevent the running back of the drum when the handle is let go.

2. A friction brake strap, D S S, which embraces a wheel with a flat rim 12 inches in diameter, and can be brought to encircle the same as tightly as we please by the lever-handle, E R D, which has a fulcrum at D. The strap, D S S, is made of steel, and, since the wheel is of considerable size, the work which can be absorbed by the friction between its rim and the strap pressing around it, as one surface slips or moves under the other, is also considerable ; in fact, the friction set up by hand-pressure on the lever at E will be sufficient to hold up or to let down gradually any weight hanging as a load upon the chain or rope wound around the drum.

132. **A clock-train** is shown in the diagram, in order to exhibit the manner in which a train of wheels is arranged in an ordinary clock.

The names given to the several wheels are marked in the sketch, and it will be seen that the minute-hand is attached to the first pinion in gear with the great wheel. Upon the axis of the great wheel is a drum, round which is coiled a cord or light chain or wire, carrying the weight which keeps the clock going by continually urging the great wheel to turn round ; and were it not for the check put upon the train by the anchor escapement and pallets there would be no measure of time. It often happens that a clock-train is used in small pieces of mechanism as a mover or motor of the wheelwork, in which case the weight is replaced by a spring coiled in a barrel, and a fusee is fastened to the axis of the great wheel. In such a train there is no scape-wheel, but the regulation of the speed is effected by placing on the last wheel of the train, or that which turns most rapidly, a small arm with vanes at each end, whereby the resistance of the air comes into play, and the necessary drag upon the wheelwork is supplied.



It is also quite common to employ a spring barrel and fusee for driving the wheelwork of an ordinary clock; in fact, the use of weights has almost died out, the spring possessing so many advantages in practice. Looking at the train, it must be arranged that the minute-hand shall go round once in an hour, and it is usual in a clock with a seconds' pendulum, to put 30 teeth on the scape-wheel, when the pendulum will make 60 swings in the period of one minute. It follows that the scape-wheel will rotate once in one minute, and if there be a seconds' hand (which is unusual in a clock) it will be attached to the arbor of the scape-wheel. The student will note that upon each axis of the train after the great wheel there is placed the combination of a large wheel and a pinion, as in the train discussed in Art. 130. It only remains, therefore, to arrange the numbers of teeth in such a manner that the scape-wheel shall rotate 60 times while the centre wheel, carrying the minute-hand, goes round once. Let each pinion, namely, those on the axis of the second-wheel and the scape-wheel, have 8 teeth, and let the centre wheel have 64 teeth, and the second-wheel 60 teeth, then

$$\text{value of train} = \frac{64 \times 60}{8 \times 8} = 60.$$

The number of teeth on the great wheel is not material, but the great wheel may conveniently go round once in 12 hours, whereby, if the pinion on the centre wheel has 8 teeth, the great wheel should have 96 teeth, and then the number of coils of the cord or chain on the barrel will determine the number of days during which the clock will keep time without being wound up. If there be 16 turns made in winding up the weight the clock will of course go for 8 days.

The mode of connecting the anchor pallets with the pendulum has already been explained.

Another clock-train might be the following :—

Let the great wheel make 26 turns and have 120 teeth,
the centre wheel and pinion 72 and 16 teeth respectively,
„ second „ „ „ 60 „ 12 „ „
scape-wheel pinion . 12 teeth.

Then

$$\frac{72 \times 60}{12 \times 12} = 6 \times 5 = 30.$$

Hence this train would work with a 2-seconds' pendulum, and is suited for a large turret clock.

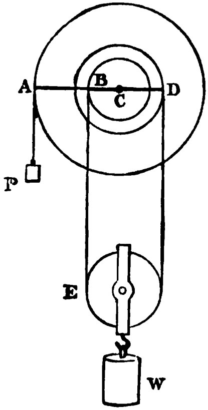
Also, when the great wheel makes 26 turns the centre pinion goes round

$$\frac{26 \times 120}{16} \text{ or } 195 \text{ times.}$$

Hence the clock will go for 8 days of 24 hours each, with 3 hours to spare.

133. **The wheel and compound axle** differs from the ordinary wheel and axle in having an axle made in two parts, the diameter of one part being made a little smaller than that of the other part. A surplus loop is left in a cord

or chain, and the two parts of it are wound in opposite directions, one on each axle, and the ends are then fastened to the respective axles. A pulley is then hung in the loop of the chain or cord to which is attached the weight to be raised. The wheel often takes the form of a lever handle, or it may be any simple equivalent for a lever, and the peculiarity of the contrivance is that the weight is being continually let down by the unwinding of the rope on the smaller axle, but is raised by the winding up of the rope on the larger axle. This is therefore called a *differential* combination, the weight rising by the difference of two opposite movements, the greater preponderating.



Then drawing the horizontal line ABCD, let T be the tension of the cord round the pulley, and we have

$$\text{P} \times \text{CA} + \text{T} \times \text{CB} = \text{T} \times \text{CD},$$

$$\therefore \quad \text{P} \times \text{CA} = \text{T}\,(\text{CD} - \text{CB}).$$

$$\text{Also} \quad 2\,\text{T} = \text{W}, \text{ or } \text{T} = \frac{\text{W}}{2},$$

$$\therefore \quad \text{P} \times \text{CA} = \frac{\text{W}}{2}\,(\text{CD} - \text{CB}).$$

Let $\text{CA} = a$, $\text{CD} = \text{R}$, $\text{CB} = r$, then

$$\text{P} = \frac{\text{W}}{2\,a}\,(\text{R} - r).$$

Ex. : Let $\text{R} = 3$, $r = 2$, $a = 12$,

$$\text{Then P} = \frac{\text{W} \times 1}{2 \times 12} = \frac{\text{W}}{24}, \quad \text{or W} = 24\,\text{P}.$$

By the principle of work, when the wheel makes one revolution,

P goes down through $2\,\pi\,a$,

E is raised through $2\,\pi\,\text{R}$ and let down by $2\,\pi\,r$;

Therefore W rises through $\frac{1}{2} \times (2\,\pi\,\text{R} - 2\,\pi\,r)$,

or W „ „ $\pi\,(\text{R} - r)$.

$$\text{Hence } P \times 2\pi a = W\pi(R - r),$$
$$\text{or } P \times 2a = W(R - r),$$
$$\text{or } P = \frac{W}{2a}(R - r);$$

the same result as before.

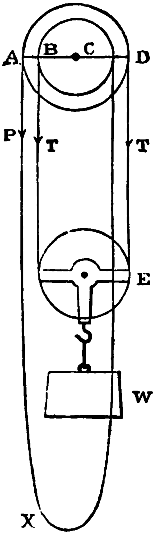
Theoretically, this is an excellent contrivance, but in practice it is most inconvenient, the reason being that a large quantity of rope must be provided while the weight rises through a comparatively small space.

Ex. : Let $R = 11$, $r = 10$, then the amount of rope coiled on the large barrel $= 2\pi \times 11 = 22\pi$, whereas the weight rises through a space $\pi\ (11 - 10)$ or π. Hence 22 feet of rope are expended when W rises 1 foot.

134. **Weston's pulley-block** is an improved form of the compound wheel and axle which is in common use, the difficulty about the expenditure of rope having been overcome by the simple expedient of employing an endless rope or chain, which merely rides without slipping on the upper surface of each pulley.

It is shown in the sketch in front elevation, and there is no coiling of the chain round each pulley, but a surplus portion hangs down as at X.

Let A B C D be a horizontal line, and T, T the tensions of the two parts of the chain passing round a single movable pulley, E. Also let the power P be just competent to support the weight W, friction being neglected.



$$\text{Then } P \times AC + T \times CB = T \times CD,$$

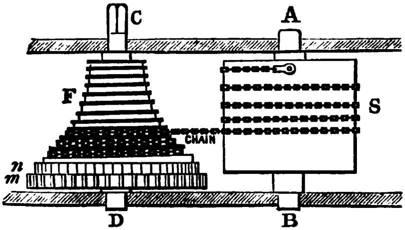
which is the equation obtained in the previous article. Hence, as before,

$$P \times AC = T(CD - CB) \text{ and } 2T = W,$$
$$\text{or } P = \frac{W}{2AC}\ (CD - C).$$

In this case, however, $AC = CD$; wherefore, if $AC = CD = R$, and $CB = r$, we have

$$P = \frac{W}{2R}(R - r).$$

135. **The fusee** is adopted in chronometers, in most English watches, and in many clocks, in order to keep up a uniform pressure on the train of wheels, and to compensate for the decreasing power of the spring. A long coiled band of tempered steel is enclosed in a barrel, whose axis is A B, and sets the wheels in motion by the aid of a cord or chain wound partly upon the barrel and partly upon a sort of tapering drum called a fusee, and marked F in the diagram.

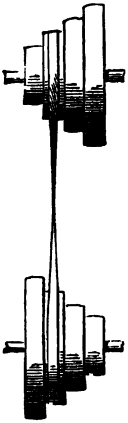


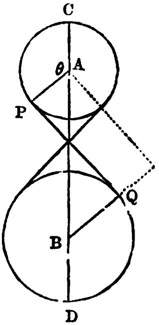
As the spring tends to uncoil in the barrel, the pull on the chain diminishes, and the object of the contrivance is to maintain this pull at a uniform intensity. The spring is wound up by a key fitted on to the winding square, C, and when the spring is fully coiled up, the greater part of the chain is wound upon the fusee, F. The wheelwork of the watch or clock is driven from the spur-wheel marked *m*, and just above *m* is a ratchet-wheel, *n*, engaging with a detent, which prevents the fusee from running back.

As the spring tends to uncoil, it exerts a pull upon the fusee, which is constantly becoming a little less powerful, also the fusee slowly turns as the watch runs down, and the chain becomes unwound from the fusee, and is rewound automatically upon the barrel. At the same time the pull of chain is felt on a part of the fusee which is nearer the base, and where the leverage is greater. It follows that, if the form of the fusee be properly adjusted to the decreasing power of the spring—which is done by trial, or should be so done—the pull of the chain will remain very nearly constant during the whole time of action.

It is easy to test the adjustment of the strength of the spring to the form of the fusee by fixing a light lever to the winding square, and trying whether or not the pull of the spring is balanced in every position by the same weight hung at the end of the lever. The fusee should be cut away a little where it might be necessary to do so.

136. **Speed pulleys** are so called because they allow of the transfer of different velocities of rotation from one shaft to another: they are much used in engineers' factories, and an example is seen in the drawings of the lathes given in the next chapter.





They are made in a series of steps, as shown, one pulley being the counterpart of the other, but pointing in the opposite direction. If the steps be equal, as is commonly the case, the sum of the radii of each pair of opposite pulleys will be a constant quantity.

There is a geometrical proposition connected with speed pulleys, namely, that when the belt connecting them is crossed, as in the diagram, and is made of the proper length, it will also be of the right length for every pair of the series. This is only strictly true when the band is crossed. In practice, open bands which do not cross are preferred, the deviation from the exact law above stated being so small that it may be disregarded.

To prove the proposition.

Let A and B represent two pulleys whose radii are AP and BQ, and assume that AP + BQ is always the same for every pair.

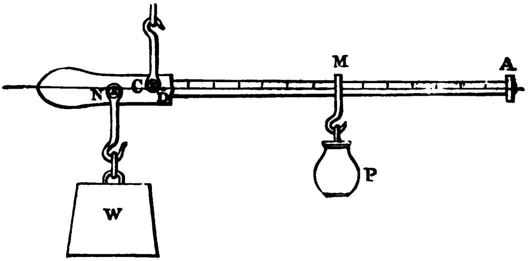
Then if AP be increased by (say) 1 inch, and BQ be decreased also by 1 inch, it is clear that PQ will remain parallel to itself, and will be shifted through 1 inch. Hence the angles, CAP and DBQ, will remain unchanged.

Then length of belt $= 2\,AP \times \text{angle } CAP + 2\,BD \times \text{angle } DBQ + 2\,PQ$

$$= 2\,(AP + BD) \times \text{angle } CAP + 2\,PQ.$$

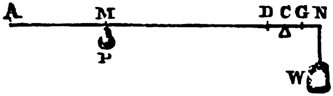
But $AP + BD$ is constant, PQ is constant, and so is the angle CAP, ∴ the length of the belt remains unchanged.

137. **The steelyard** is a simple unbalanced lever having a fulcrum near one end. It is shown in the sketch, where the fulcrum, C, rests in the loop of a hook. The weight, W, is suspended at a point, N, and a movable weight, P, hanging on a ring, M, is placed at such a distance along the graduated arm, CA, as just to balance W.



The difficulty connected with the steelyard is the method of graduating it, for no attempt is made to balance the steelyard so that it may rest in a horizontal position when P and W are removed.

Remove P and W, and let Q be the weight of the steelyard, with the hook at N after P and W are taken off. We shall suppose that the weight, Q, acts as if it were collected at a certain point, G, and in order to counterbalance this weight we thread P on the arm CA and move it to



a point, D, to be found by trial, such that P at D balances Q at G, the point C being the fulcrum.

$$\text{Hence } P \times CD = Q \times CG.$$

Now hang W at N, and let it be counterbalanced by P at M. We must remember that there is a permanent weight, Q, acting at G.

$$\begin{aligned} \text{Therefore} \quad P \times CM &= W \times CN + Q \times CG \\ &= W \times CN + P \times CD. \end{aligned}$$

$$\begin{aligned} \text{Hence} \quad P\,(CM - CD) &= W \times CN \\ \text{or } P \times DM &= W \times CN. \end{aligned}$$

$$\text{Let} \quad W = 1 \text{ lb. then } DM = \frac{CN}{P}$$

$$W = 2 \text{ lb.} \quad \text{,,} \quad DM = \frac{2\,CN}{P}, \text{ and so on.}$$

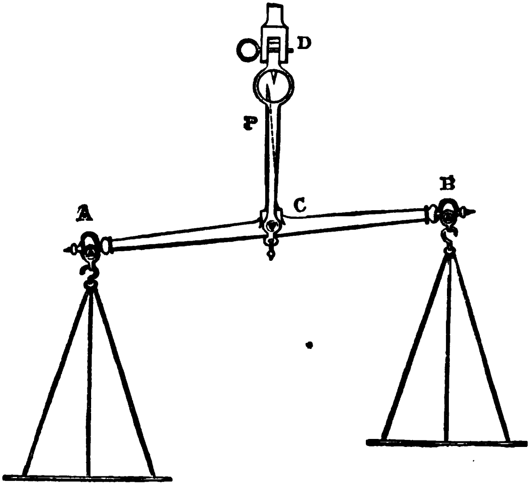
It appears, therefore, that the graduations 1, 2, 3, 4, &c. lbs. will reckon from the point D, and not from the fulcrum C.

The point D is marked in both diagrams, and it will be seen that the graduations measure from that point.

138. **The balance** is an instrument of the highest importance, and is constructed in a variety of forms, according to the uses to which it is applicable. It consists essentially of a lever, A B, with equal arms, supported on a fulcrum, C, and having scale-pans suspended at the ends of the arms. The drawing on page 144 is taken from a small instrument of fairly good construction, such as is commonly met with, the length of the beam being eight inches. When the weights in the two pans are equal, the arm rests in a horizontal position, and when one weight is a little in excess, the beam tilts through an angle, as indicated by the pointer P, shown in the sketch. The balance hangs from an arm, D, which can be raised or lowered at pleasure.

Since the beam of the balance is essentially a straight lever, the first thing to provide for is that the point of support of the beam, and the points from which the scale-pans hang, shall be in the same straight line.

It is the universal practice to adopt, for the support of the beam and the suspension of the scale-pans, a system of so-called knife edges, which are triangular wedges of steel resting upon hardened steel or agate surfaces. Such knife edges are shown in the sketch, and again in the next diagram, the angular point of each wedge being in the straight line A C B.



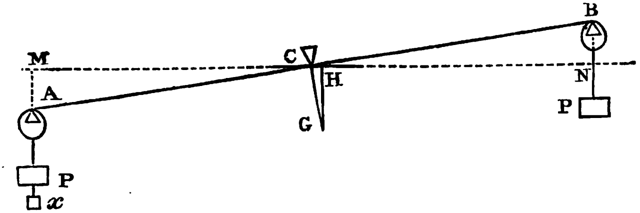
Also it is essential that the beam should rest in stable equilibrium on the point C, and its centre of gravity must therefore lie below the point C.

Where it is required that the balance shall indicate small differences of weight, the centre of gravity of the beam should be brought very near to C.

139. **Sensibility of balance.**—The balance, as employed in a chemical laboratory, is an instrument containing so many refinements of construction that it is not possible

to describe it within the space at our disposal, and there is only one matter which can be touched upon, viz. its *sensibility*, which is measured by the deviation of the beam from the horizontal position is caused by a given small excess of weight in one of the pans.

Let A C B be the beam of a balance, C the point of suspension, W the weight of the beam and its appendages acting at their centre of gravity, G. Let there be a weight, P, in one scale-pan, and $P + x$ in the other pan.



Draw the horizontal line M C N through C, meeting the verticals through A and B in M and N, and draw G H perpendicular to M N.

Then $(P + x)\,CM = P \times CN + W \times CH$.

But $CM = CN \quad \therefore \quad x \times CM = W \times CH$.

$$\text{But by similar triangles, } \frac{CH}{CG} = \frac{AM}{CA}$$

$$\therefore \quad x \times CM = \frac{W \times CG \times AM}{CA}$$

$$\therefore \quad AM = \frac{x \times CA \times CM}{W \times CG}$$

But $CM = CA$ very nearly,

$$\therefore \quad AM = \frac{x \times CA^2}{W \times CG}.$$

It is evident that A M is a measure of the sensibility, and that the greater A M is, the more sensitive will be the balance for any given difference of weights in the two pans.

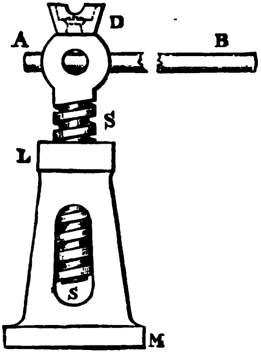
Also the sensibility is increased by increasing C A, by diminishing W, and by diminishing C G.

Ex.: Let CA = 10 inches, CG = $\frac{1}{20}$ inch, x = $\frac{1}{2}$ grain, W = 2,000 grains.

$$\therefore \quad W \times CG = \tfrac{2000}{20} = 100.$$
$$x \times CA^2 = \tfrac{1}{2} \times 100 = 50.$$
$$\therefore \quad AM = \tfrac{50}{100} = \tfrac{1}{2},$$

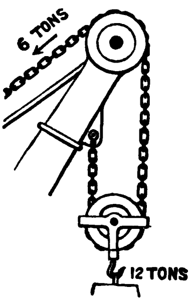
or the point A would descend $\frac{1}{2}$ inch for a difference of $\frac{1}{2}$ grain in the two pans.

140. **A lifting-jack,** which frequently takes the form of a screw-jack, is shown in the diagram. It here consists of a strong square-threaded screw, S S, passing through a nut or portion of a female screw at the top of a strong casing, L M, which serves as a pedestal to support any load resting on the screw-jack. The screw, S S, is rotated by a lever handle or bar, A B, passed through a hole in an enlarged head at the top of S S.



The weight to be raised rests on a claw, D, which can rotate freely in a socket, whence it follows that the rotation of the screw will cause the body resting on D to rise or fall without any rotation.

141. **The snatch-block** is an application of the single movable pulley which is often seen in a crane, and is shown in the drawing, taken from a lecture diagram by Sir J. Anderson. The end of the jib of the crane with the so-called snatch-block, or single movable pulley, forms the subject of the sketch, the weight supported being twelve tons, and the pull of the chain being reduced one half, namely, to six tons, by the use of the block or pulley.



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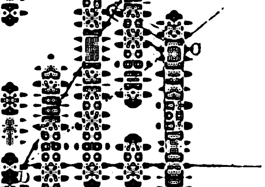

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CHAPTER X.

TRUTH OF SURFACE—STRENGTH OF MATERIALS—THE LATHE.

IT has been said by an eminent mechanician that the first lesson which the student of mechanics ought to learn is the method of producing true surfaces. The power of constructing machinery of precision depends on the excellence of machine tools, and such an instrument as a Whitworth lathe has become proverbial for the accuracy of the work which can be turned out from it. As the beginner advances in his knowledge he will find that the great point to be aimed at in practice is truth of surface, and a few observations, which are merely suggestive, may here find a place.

The two primary surfaces of essential importance in the workshop are a so-called 'true plane' and a cylinder. The first may take the form of a *surface plate* made in cast iron, the second may be a gauge or cylinder of steel a few inches in length, with a collar to fit, and so accurately worked to truth that a difference of $\frac{1}{10000}$ of an inch in size between the cylinder and the collar shows a difference in the fit which must surprise any one who sees it for the first time. It is something also to be able to measure, without any magnifying-glass, a difference of $\frac{1}{10000}$ of an inch, and the power of doing this depends on a measuring-machine, wherein truth of surface is essential.

144. **A surface plate,** upon which the engineer depends for the truth of plane surfaces, was first brought into notice by Sir J. Whitworth in 1840, and originated from a systematic attempt to bring the plane surface on the bed of a lathe to the highest perfection.

Up to that time the process relied upon for obtaining plane surfaces, and, indeed, the only one practically used, had involved the grinding two plates together with emery powder and water. While the operation was going on the plates under preparation were occasionally compared with a standard plate, but the standard itself was imperfect, the method of comparison was uncertain, and the results now arrived at could not be approached.

The operation of grinding fails for the obvious reason that the action of the powder cannot be restricted to those parts only which are in error, and, accordingly, Sir J. Whitworth proposed to obtain truth of surface by a system of scraping away all the parts which are out of truth.

The surface plate, as commonly used, is a rectangular plate of cast iron, strongly ribbed at the back, and standing upon three bearings or supports. In a more recent form it is a hexagonal plate having three bearings in the points of an equilateral triangle. It was called by the inventor a *true plane*, and, although absolute truth is impossible, it presents a wonderfully close approximation to exact truth.

In order to originate a standard surface plate, it is necessary to operate upon *three* plates at the same time. Call them A, B, and C, and let them be prepared in the planing-machines by levelling the supports and planing over the surfaces. A little red ochre and oil being smeared on plate A, the plates B and C are rubbed upon it in order to take up the colouring matter where there is contact. The parts so marked are above the general level, and are scraped by a small scraper of steel having two facets at an angle of 60°. In this way B and C are brought to coincide with A; then B and C are compared together, and one of them, as B, is corrected, after which A and C are brought to coincide with B, and so on until there is a perfect coincidence between all three plates, when each must be a plane surface.

The standard being obtained by scraping, its surface is mottled throughout by the action of the tool, and it consists of an assemblage of bearing surfaces which become bright by the rubbing that takes place during comparison, and which should be closely and evenly distributed throughout its surface.

We may sum up the conditions required as follows:

1. The bearing surfaces should all lie in the same plane.
2. They should be distributed as nearly as possible at equal distances from each other.
3. They should be sufficiently numerous for the particular application intended.

A standard plate being once obtained, it can be used for reference in the workshop, and by the system of scraping any other surface may be brought up to the same accuracy as the standard to which it is referred.

From the true plane we pass on to a measuring-machine capable of measuring the exact length of a rectangular bar with plane ends perpendicular to its axis, and thus we obtain standards of one foot, one inch, &c. From plane surfaces we pass to cylinders, and obtain cylindrical gauges, which complete the measuring apparatus.

With the improved machinery of the present day, a manufacturer can reproduce any number of pieces which shall be interchangeable, and form, when put together, a complete instrument, although each separate part is picked out from a number of others of the same kind.

145. **Stress and strain.**—In common language, these words are used indifferently to express either the force acting on a body (say a bar of iron) to bend or break it, or the extension and amount of change of form whereby it becomes bent or otherwise affected by the force in action.

Strain is, however, defined in technical treatises to be the alteration of figure, of whatever kind, produced in a body by the operation of force.

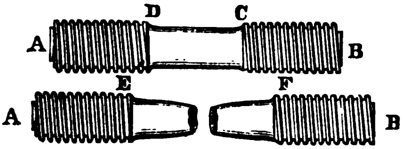
Stress is the force producing such change of form.

146. **Tensile stress** occurs when a rod or bar is

securely clamped at one end and supports a given weight hung upon it. The effect of the load is simply to stretch the bar, or, in extreme cases, to pull it asunder into two pieces.

The metals used by engineers, such as iron, steel, gun-metal, &c., are carefully tested in machines made for the purpose, whereby the exact stress can be accurately measured.

For example, Sir J. Whitworth has tested the strength of steel as follows :—A B is a cylinder of steel, two inches in length in the central part D C, where it is carefully turned so as to be quite cylindrical, and is ·7979 inch in diameter, the area being ½ of a square inch. The specimen here shown is screwed into blocks, but a test-piece has usually enlarged ends, whereby it is held when placed in the machine, and it is subjected to a continually increasing pull, which is measured. It soon yields and lengthens a little, after which, if the stress be taken off, it will recover its original dimensions. But if the pull exceeds a certain limit the test-piece extends or elongates permanently, it becomes of smaller diameter in the central part, and may lengthen to the extent shown in the diagram before it finally breaks into two pieces.



If the fractured pieces are fitted together, the length of the part E F as compared with D C will show the amount of elongation. Thus, if E F be 2½ inches and D C be 2 inches, we should call the elongation, or ductility, 25 per cent.

147. The laws observed by experiment are the following :—

(1) The amount of extension for the same bar is, up to a limit, in direct proportion to the stretching weight.

(2) It varies as the length of the rod.

(3) It varies inversely as the sectional area of the rod. That is, if the area be doubled, the resistance to extension will also be doubled,

(4) It depends on the nature of the material. Thus good steel resists extension more powerfully than wrought iron.

The student will find tables in which the tensile strength of materials is given for a bar 1 square inch in sectional area. From such a table the tensile strength of any other sized bar can be inferred by rule 3. The breaking weights are in direct proportion to the cross sectional areas.

148. **The limit of elasticity** is a phrase applied to denote that the test specimen has been stretched to the utmost limit at which its recovery of form is perfect when released from the machine. To observe this limit, two fine cross lines are engraved on D C by a scriber, and the distance is carefully noted, or an indicator of some kind may be used. The bar is put under tension and the load is taken off, the distance between the lines being gauged. As soon as permanent elongation occurs, the limit of recovery of form, or the limit of elasticity, is reached.

Thus the limit of elasticity of wrought iron is about 10 tons on the square inch.

But the bar will not break with a load of (say) 15 or 20 tons. It would, however, be unsafe to load it to that extent, as fracture might occur at any time.

In trials with the testing-machine the bar would bear (say) 24 tons without absolutely giving way. With that load, however, it would go on elongating and would finally break. Hence there is a record taken of

(1) The limit of elasticity.

(2) The breaking weight.

(3) The ductility or proportionate elongation.

149. **The modulus of elasticity** is a term used to express the ratio $\frac{\text{stress}}{\text{strain}}$ in the case of a rod of any material subjected either to a tensile strain, or to compression, within the limits of elasticity.

Here the stress will be expressed in pounds per sq. inch of section, and the strain will be the elongation in inches per inch of length of the rod.

For example, let a rod of iron 10 feet long, and 1 sq. inch in section, be elongated by ·0052 of an inch under a stress of 1262 lbs. Then, since the rod is 120 inches in length, the elongation per inch is $\frac{\cdot 0052}{120}$. Therefore

$$\frac{\text{stress}}{\text{strain}} = \frac{1262}{\frac{\cdot 0052}{120}} = \frac{1262 \times 120}{\cdot 0052} = 29{,}120{,}000 \text{ lbs. per sq. in.}$$

If it were possible for the rod of sectional area, 1 sq. inch, to elongate to twice its length without exceeding the limits of elasticity, we should have *strain* = 1 inch, in which case the modulus would be the number of pounds necessary to produce this result, which is an enormous number in the case of the materials used by engineers, as appears from the result given above.

150. **Cast iron** is the form in which iron first appears when manufactured in blast furnaces from ironstone ores.

It is the metal iron impregnated with a considerable quantity of carbon derived from the fuel employed to melt it, the effect of the carbon being to render the iron both fusible and brittle.

Cast iron is often known as *pig iron*, being cast in small bars called 'pigs,' and the student should understand that the combination of iron with carbon is matter for careful study, because the mechanical qualities of the metal are so largely influenced by the presence of carbon.

Iron will combine with carbon in chemical union, in which case the metal presents certain well-defined properties, but it will also take up carbon in the form of *graphite* or blacklead (from which pencils are made), the graphite being mechanically mixed with the iron, and then the metal presents another set of properties. The student must refer to treatises on metallurgy for information on this complex subject, as he will here find merely an outline of two or three prominent points.

Cast iron may contain from 2 to 5 per cent. of carbon, and if the whole of the carbon were combined with the iron the product would be *white iron*, which is silvery in fracture, and so hard that it cannot be shaped by cutting tools. It is therefore not used for casting purposes.

If some 2 to 3 per cent. of the carbon be present in the form of graphite the iron changes its character ; it becomes *grey* in fracture, is very fluid when molten, so that it penetrates easily into the intricacies of a mould, and, what is of great importance, it can be shaped by cutting tools in the workshop. It is friable and breaks up under the cutter, and that is why it is selected for a surface plate. The scraper removes the metal in a fine dust.

The average specific gravity of cast iron is 7·1. The average weight of a cubic foot is 444 lbs. and of a cubic inch, ·257 lb.

Iron contracts in cooling when cast, the contraction for large castings being $\frac{1}{8}$ inch per foot, and in small castings $\frac{1}{16}$ inch per foot.

The tensile strength of cast iron is given at 7 or 8 tons per square inch, and the resistance to compression is about six times as great—a property which is taken advantage of in designing cast-iron beams.

According to Sir J. Anderson, the limit of elasticity is about half the breaking weight.

A cast-iron test-piece does not elongate perceptibly to ordinary observation, and it suddenly breaks without giving any warning.

In practice the safe load in tension for cast iron would be from $1\frac{1}{2}$ to $2\frac{1}{2}$ tons per square inch.

151. **Wrought or malleable iron** is the metal nearly pure, or not contaminated with so much as $\frac{1}{5}$ per cent. of carbon. It may be forged and welded, and is very ductile. The fracture is fibrous, but in some cases crystalline. It

does not melt, but merely softens in an ordinary furnace, and is shaped into useful forms by forging and compression, but not by moulding. Dilute nitric acid leaves a green stain upon it. Sir J. Whitworth has found the tensile strength of the best Yorkshire or Lowmoor iron to be 27 tons per square inch, with a ductility of 38 per cent. The specific gravity of Yorkshire iron is given as 7·76, the weight of a cubic foot at 485 lbs., and of a cubic inch at ·281 lb.

As an average, the elastic limit of Lowmoor iron may be taken at 26,500 lbs. per square inch in tension and 26,000 lbs. in compression, the breaking load being 54,000 lbs. when in tension.

As a practical rule, the safe load for good wrought iron would be 4 tons per square inch in tension and 5 tons in compression, or (say) 4½ tons for both.

152. **Steel** is iron combined with a very small percentage of carbon, its distinguishing property being that it becomes hardened by sudden cooling. Wrought iron has no such property, but becomes *steely iron* on acquiring 2 or 3 parts of carbon to 1,000 of the metal.

The steel employed for rails or armour plates contains the smallest amount of carbon, that for cutting tools may contain 1 to 1½ per cent. of carbon.

When heated sufficiently, steel may be forged, and if allowed to cool very slowly it remains soft, like malleable iron, and can be shaped by cutting tools with great facility.

But a piece of soft steel may be readily hardened, for if it be raised to a dull red heat and suddenly cooled by being plunged into oil or water, it becomes extremely hard and brittle; so hard, indeed, that no tools (which are themselves made of hardened steel) can touch it.

This is one of the valuable qualities of steel. For example, in coining sovereigns a steel die is engraved, and is then hardened and impressed on a piece of soft steel. This makes an exact counterpart in relief. The counterpart is hardened, and may impress a copy of the original engraved die on a number of pieces of soft steel; each of these may be hardened and used for stamping the disc of gold which is to form a sovereign.

In order to make this hardened steel less brittle, and yet preserve sufficient hardness for the cutting edge of a tool, the steel is *tempered*, or let down, as it were, from the extreme state by heating to a certain temperature and then allowing it to cool slowly. If any part of the surface be bright a thin film of oxide forms thereon, and the colour changes with the gradations in temperature from pale straw to brown yellow, and so to a purple passing into a dark blue, such as we see in watch-springs.

The yellow varies from 430° to 490° for lancets and cutting tools.

The brown, yellow, and purple from 500° to 530° for hatchets, chipping chisels, &c.

The blue from 550° to 600° for springs, swords, &c.

Or the hardening and tempering may be done at one process by heating the steel to a temperature less than red heat, and then plunging it into a bath of oil and allowing it to cool therein. When steel is tempered its limit of elasticity may be as much as three times that of the wrought iron from which it has been manufactured.

Steel may be cast, but under ordinary conditions the metal may suffer from the presence of small gas-cells. Sir J. Whitworth has patented a process of manufacturing steel for engineering purposes by compression when in a liquid state, and he subjects the liquid steel to a pressure of from 10 to 20 tons on the square inch. Steel so treated is known as *fluid compressed steel*, and has great strength as well as ductility, being uniform in texture throughout. One quality, produced in large quantities, has a tensile strength of 40 tons per square inch, with a ductility of 32 per cent.

Specimens of steel may often have a tensile strength of 50 to 60 tons per square inch.

It is stated that mild steel, with a tensile strength of

30 tons and a ductility of 26 per cent., would probably have a limit of elasticity of about 13 tons.

Dilute nitric acid makes a dark grey stain upon steel.

153. **Copper** is a most valuable metal, but is used by engineers more commonly when alloyed with other metals than in a separate state. It can be forged, hence the use of copper bolts, but its tenacity is inferior to wrought iron, as it will break under a stress of 15 tons per square inch. In the ingot or cast condition it is very much weaker than after hammering.

Its specific gravity is 8·8, the weight of a cubic foot being 550 lbs. and that of a cubic inch ·318 lb.

Tin is principally used by engineers in alloys with copper. It protects iron plate as a covering, and the iron is then called 'tin plate.' It is used with some admixture as a lining for brasses, when it is called 'white metal.' It is very soft, and its tensile strength is 2·1 tons per square inch. It melts at 442° Fahr. Its specific gravity is 7·3.

Brass is an alloy of copper and zinc, called C and Z, and is $2C+Z$, which proportions may be varied. It is of a yellow colour, and is very easily worked in the lathe. When melted some of the zinc always burns away and gives rise to the flame which is observed in pouring brass into a mould. It cannot be forged, being rotten while hot.

Muntz metal is $3C+2Z$. It is very strong, and is valued accordingly. Its tenacity is about 22 tons per square inch, being nearly equal to that of good wrought iron. It is very ductile, and can be forged when hot. By the addition of about 1 per cent. of tin, called T, it becomes naval brass, and can resist the action of salt water.

Gun metal is $9C+T$. It has a much redder tinge than ordinary brass. Its average tenacity is given by Sir J. Anderson as 31,280 lbs. per square inch, and its limit of elasticity is 15,164 lbs. per square inch, its ductility being 25 per cent.

154. **Transverse stress** upon a beam or bar is pro-

duced by a force acting perpendicularly to the axis of the bar.

There are two principal cases :

(1) When a bar or beam is fixed at one end, or built into a wall and stands out perpendicularly for supporting a load. It is then called a *cantilever.*

(2) Where a bar or beam is supported at both ends, and loaded in the intermediate portion, as in the case of the joists or beams of timber which support the flooring of a room.

The problem of finding the form and dimensions of a beam in order to resist a given transverse stress is divided into two parts.

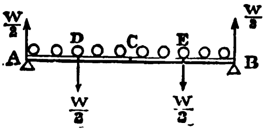
First. We calculate the moment of the forces arising from the weight of the beam and the load upon it, and estimate that moment about any point of the beam which may be selected.

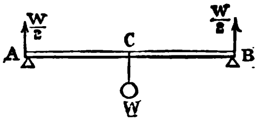
We thus estimate the so-called *bending moment,* or the measure of the effect of the forces tending to bend or break the beam, at any point of it.

As an example, take two simple cases :—

(1) Let A B represent a rectangular beam of length, l, which is supported at its ends and loaded in the centre, C, with a weight, W. Then $\frac{W}{2}$ is the pressure at A or B, the weight of the beam being neglected. Hence

$$\text{Bending moment at C} = \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4}.$$





(2) Let A B represent the same beam with the same load, W, uniformly distributed along its length as represented

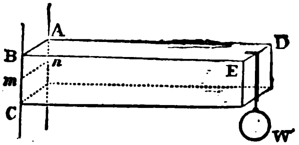
by a number of small loads. Then the load on C B is $\frac{W}{2}$ acting at the middle point, D. Therefore

$$\text{Bending moment at C} = \frac{W}{2} \times \frac{l}{2} - \frac{W}{2} \times \frac{l}{4} = \frac{W\,l}{8}.$$

This shows the influence of the distribution of load.

Secondly. We assume that the beam is made up of a number of elastic bars or fibres running parallel to its length. Conceive that the beam is fixed into a wall at one end and loaded at the other end, and it will be apparent that the fibres in the upper part will be in tension and those in the lower part will be in compression. Somewhere in the middle of the beam the fibres will be neither extended nor compressed. There is a mathematical investigation beyond the student at present, which gives the sum of the moments of the resistances of all the fibres to extension and compression, such moment being taken about the point where there is neither extension nor compression. After this moment has been ascertained, it only remains to equate it to the bending moment produced by the external forces, and the problem can be fully solved.

In order to give some idea of what is meant, take the case of a *rectangular* beam, B D, standing out horizontally from a vertical wall, and loaded at one end by a given weight, W.



Let A B $= b$, B C $= d$, A D $= l$, and assume that $m\,n$ represents the line about which the moments of resistances of the fibres are taken.

Then theory tells us that the sum of the moments of the resistances of the fibres $= \text{S}\,b\,d^2$, where S is a number dependent on the material of the beam. Also the bending moment $= \text{W}\,l$, in this case,

$$\therefore\ \text{W}\,l = \text{S}\,b\,d^2,$$

an equation which enables us to solve numerical examples. Again, if the beam had been supported at both ends and loaded in the centre with a weight, w, we should have had the equation

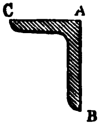
$$\frac{\mathrm{w}\,l}{4} = \mathrm{s}\,b\,d^2, \text{ and so for other cases.}$$

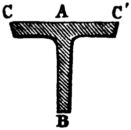
The quantity s being thus known for any particular material, a variety of questions may be proposed on the formula

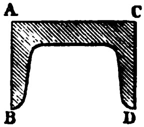
$$\frac{\mathrm{w}\,l}{4} = \mathrm{s}\,b\,d^2$$

or upon other like formulæ. Thus if the dimensions of the beam are given, we can find the weight which it will support, or if w, l, and b are given, we can find d, and so on.

155. Having proved that the strength of a rectangular beam varies as (breadth) × (depth)^2, we should endeavour to dispose the material of an iron beam in a more advantageous manner than is practicable with timber. There are three common forms of rolled bar iron used in construction, which are known as *angle iron*, *T iron*, and *channel iron*, where the strength due to increased depth is exhibited.





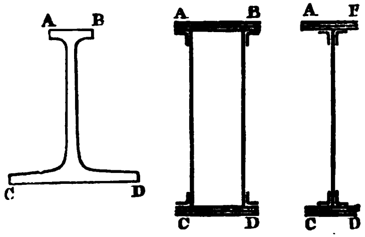


They are placed in order in the sketch. In the case of angle iron, there is increased strength due to the increased depth in the two perpendicular directions, A C, A B. In the T iron we have a sort of double angle iron, and in the channel iron there is again the increased strength due to the parts A B, C D. The last form is much used in the construction of iron bridges, and, as often happens in mechanics, it has proved useful in very homely applications. The frames of umbrellas are now made of steel wire rolled in the form of channel iron, and are stronger and lighter than those of the old-fashioned construction, where the ribs were solid square wires.

156. In girder beams the principal forms of section are shown in the diagram.

The term *girder beam* is technically applied to beams of wrought or cast iron, such as are in use for supporting roadways or the floors of warehouses.

As stated, the resistance of cast iron to compression is 6 times its resistance to tension. Hence the material is disposed in two flanges, A B, C D, united by a web, and the area of the top flange, A B, which resists compression, may be one-sixth of the area of the bottom flange, C D, which resists extension. In practice the area of the top flange is often one-fourth that of the lower flange.



Whereas in wrought-iron beams, where the resistances to compression and extension are nearly equal, the material may be disposed in two equal flanges, A B, C D, united either by a single web, or by a double web, forming a box, as in the sketch.

The student has, no doubt, observed that in many cases an iron beam is deeper in the centre and tapers in a somewhat bowed form towards the end. This shows the influence of length. In the simple case of a beam supported at the ends, and uniformly loaded, it will be found that the bending moment increases as we approach the centre, and therefore the cross sectional area must be enlarged in like proportion. The law of distribution of the material in such

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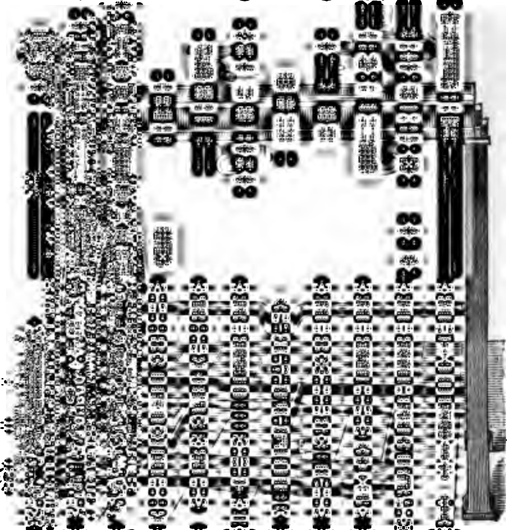


[illegible] the crank [illegible] tween the [illegible] entre, and [illegible] he treadle, [illegible] tween the [illegible]

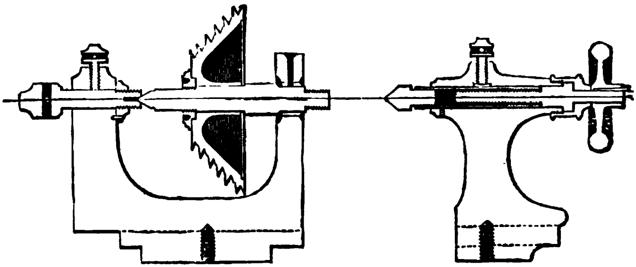
[illegible] workman [illegible], which is [illegible] he crank is rising. At this time the pressure of the foot is taken off, or the wheel would not rotate. Here is an example of the inertia of matter; the driving force, namely, the pressure of the foot, acts only during half a revolution of the fly-wheel, and the tendency of the wheel to continue in motion, according to the first law, carries it round the other half revolution. Also, when the fly-wheel is moving at a sufficient rate, it maintains a nearly uniform speed, because a considerable amount of work is stored up in it.

The fly-wheel serves another purpose, for it is in fact a speed-pulley, having grooves of different diameters upon it, and being connected with a much smaller grooved pulley at *d*, whereby different speeds may be given to the axis which carries the work, technically called the *mandril* of the lathe.

In the lathe represented in the figure, the speed of the mandril can be varied from 7 times to $1\frac{2}{3}$ times the speed of the crank-shaft. Without speed-pulleys a certain variation of rate of motion can be obtained by driving more or less vigorously with the foot, but the shifting of the band gives a series of different speeds while the foot is working at a uniform rate. Such a variation is essential, because a lathe of this kind may be used for turning hard or soft materials, or articles differing considerably in diameter, for which different speeds are required.

The drawing shows also the fixed headstock, *d*, and the movable headstock, *e*, two very important parts of the lathe, and on this account we give a separate drawing of them, which should be carefully studied.

The mandril is the short round spindle or shaft of which the hinder end is pointed into a cone, and the other end is screwed and projects through the framing. To the screwed end of the mandril, pieces called *chucks* are secured, which carry the work, and it is therefore essential that the mandril should rotate with its axis in one invariable line, and should not shake or become loose by wear. During the operation of turning it may also be subjected to considerable endlong pressure. It will be seen that, in the front bearing, the mandril is cylindrical for about $\frac{5}{6}$ of the thickness of the headstock and terminates in a conical shoulder, which is carefully faced ; also the bearing which supports the mandril is *bushed*, or lined with steel.



The hinder end of the mandril is carried by a screw, the steel end of which is slightly hollowed out to receive it, and there is a fine leading-hole running into the body of the screw for some little distance, so that any wear of the rubbing parts may not affect the true line of direction of the axis. Sometimes this arrangement is reversed, the screw being pointed and the mandril recessed. But in either case, by slightly tightening up the screw, any wear can be compensated. There is also a set screw which holds the adjusting screw perfectly firm when its position is once determined.

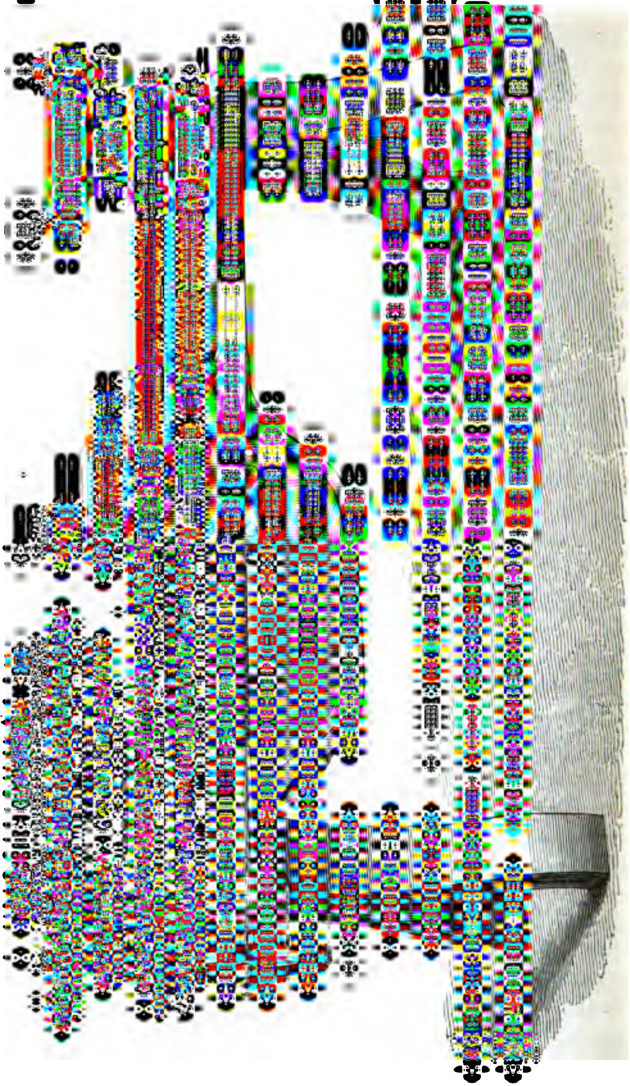
The movable headstock, *e*, or *back poppet*, is also shown in both figures. It is movable along the bed of the lathe, and is clamped from below by a screw. Its chief use is to give the additional support of a dead or immovable centre to any piece of work which would be unsteady without it, and there is to be seen within it an excellent example of the use of the screw for a traversing motion.

The steel point, or *back centre*, as it is termed, is made conical, so as to enter the work to a short distance, and to support it while rotating with as little friction as possible. The headstock being locked in position, the dead centre must be capable of being readily adjusted to the exact position required.

A small hand-wheel is keyed to a screwed spindle, shown in the drawing, which works in a hollow screwed cylinder carrying the projecting dead centre. This hollow cylinder is prevented from rotating by a pin running in a groove on its under side. We have therefore a screw and nut, but have arranged that the nut shall not rotate, but shall only move longitudinally. It follows that when the screw is turned by the hand-wheel, the nut or hollow cylinder will travel in or out, as may be required. But the dead centre is part of the sliding cylinder and moves with it, whereby the desired object is attained. There is a small set screw to lock the sliding cylinder in any given position.

158. **A screw-cutting power lathe** is shown in the diagram on page 166, and is driven by steam power brought by a belt from a shaft overhead, and running upon the stepped pulley at one end of the lathe. In a lathe of this character much of the work requires a very slow speed, and therefore there are special arrangements for the reduction of velocity, which will presently occupy our attention.

The general character of the lathe is clearly shown in the drawing. There is the bed of cast iron with its true plane surfaces, along which a sliding saddle moves as required. This saddle carries the *slide-rest*, intended to support the cutting tool, and carrying out in a perfect manner the principle of rectangular co-ordinates, explained in Art. 25, whereby the position of any point in a plane may be commanded by motions in two lines at right angles to each other. A separate drawing of the slide-rest will be given farther on. The headstock at the other end of the lathe is like that previously described in the foot lathe, being, however larger and more carefully constructed.



Just underneath the bed of the lathe there is a rack, into which a pinion, concealed within the framing of the saddle, gears; and this rack and pinion serve to shift the slide-rest rapidly along the bed. The handle for turning the pinion is shown.

Lower down, and running along the whole length of the lathe, there is a screw, called a *leading* screw, which is used for screw-cutting; and a train of wheels, such as would be employed for cutting a screw of some definite pitch, connects the leading screw with the mandril of the lathe. The student will understand that the leading screw may be connected with or disconnected from the saddle at pleasure, inasmuch as the lathe may be wanted for screw-cutting, or may be required for some other purpose. This connection is made or released by a clamp-nut, as it is termed, being two parts of a nut which can be brought down upon the screw so as to embrace it as one nut, or can be moved altogether out of the way.

159. **The double-geared headstock** requires a separate description, the annexed diagrams (p. 168) being taken from a Whitworth lathe belonging to the Normal School.

The upper diagram shows an elevation of the headstock, and the lower diagram is a plan of the same. It consists of a strong cast-iron framing bolted to the bed of the lathe, and carrying the steel axis or mandril, A B. Both the bearings of A B are conical, and may be formed on the mandril itself, or, if preferred, the hinder cone may be a loose conical bush carefully fitted to the mandril. This precaution enables an accurate fit to be maintained between the mandril and the headstock. These cones work in hardened steel bushes let into the framing in order to lessen the wear at the bearings.

The conical end, A, gives the true centre of the work supported in the lathe, and the piece, C, which is screwed on to the end of the mandril, is the driver, whose projecting arms carry the work round as the mandril rotates.

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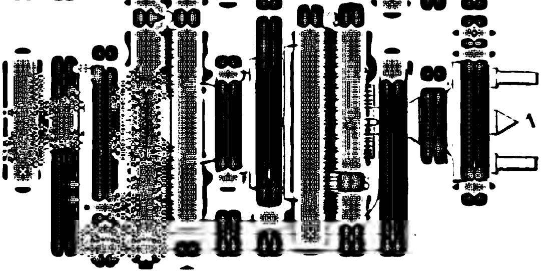

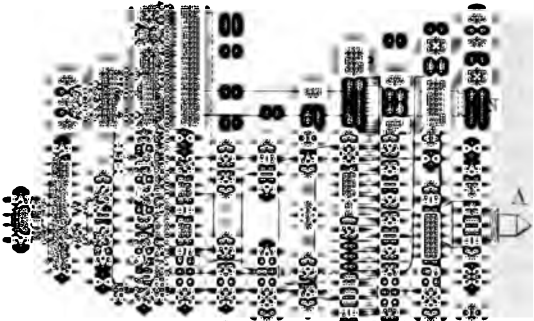

[illegible] spur-wheel [illegible] il. When [illegible] bolt and [illegible] the strap [illegible] will give [illegible] teps upon

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The shaft, M N, has two positions, one where D and P gear with E and Q respectively, the other where the wheels D and P are out of gear, and M N is lying idle.

When M N has the position shown in the drawing, the bolt S is moved so that Q is no longer locked to R, whereby the power enters at R, and passes through the successive wheels E, D, P, Q, whereof the wheel Q is keyed to the mandril. Hence, if the numbers of teeth on these respective wheels be e, d, p, q, the speed of the mandril will be reduced by the multiplier

$$\frac{e\,p}{d\,q}.$$

For example, let $e = 21$, $d = 63$, $p = 21$, $q = 63$, then the effect of the train of wheels is to reduce the velocity of the rotation of the mandril by one ninth part.

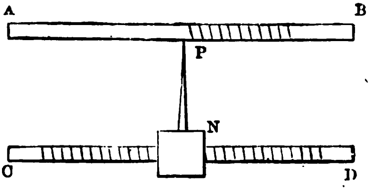
On the whole, then, there is either the reduction of speed obtained by shifting the driving-strap from the larger to the smaller of the steps on the speed pulley, or there is the further reduction due to the train of wheels, which again is modified by the successive steps on the pulley, R.

In order to lock the shaft, M N, in the two positions there are grooves upon it, one of which is marked n, and there is a pin, m, which enters the groove and prevents the shaft from sliding longitudinally, but does not interfere with its rotation.

160. The value of the leading screw, as supplied for cutting screws of different pitches, will now occupy the student.

It is here that the so-called *copying principle* receives one of its most valuable applications. The maker of a lathe furnishes a leading screw with (say) four threads to the inch, and by the use of the lathe this screw can be copied any number of times, and with any variations in the pitch which may be desired. In other words, the lathe now carries within itself a copy which can be reproduced or varied at pleasure.

An elementary diagram will make this clear. The leading screw which forms the copy is marked C D, and is a screw of a definite pitch. This screw carries a nut, N, and, disregarding the actual construction, we will suppose that the nut N is furnished with a pointer, P, capable of tracing a screw thread upon another axis, A B.



Conceive now that A B and C D are connected by a train of wheels in such a manner that they can revolve with any required relative velocities.

Upon each revolution of C D the nut advances through a space equal to the pitch of the screw. If A B also revolves at the same rate as C D, and in the same direction, the point P will describe upon A B a screw thread exactly similar to that upon C D.

If A B revolve more rapidly than C D, the pitch of the screw upon A B will be less than that upon C D, and the opposite when A B revolves less rapidly than C D.

In the drawing of the lathe there is a train shown which connects the leading screw, called C D, with the mandril, called A B.

Let C D have two threads to the inch, and let the successive wheels from the leading screw to the mandril have 60 teeth, 25 teeth, 50 teeth, and 30 teeth, then

$$\frac{\text{vel. of A B}}{\text{vel. of C D}} = \frac{60 \times 50}{25 \times 30} = 4$$

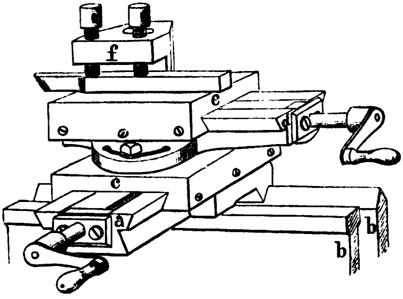
or, when C D makes one turn, A B makes four turns.

It follows that the number of threads per inch on A B $= 2 \times 4 = 8$, and with this train of wheels the lathe will cut a right-handed screw with eight threads to the inch.

It is usual to supply a set of change wheels with screw-cntting lathes, and a table indicates the wheels required for cutting a screw with any given number of threads to the inch. An example has already been given in Art. 129.

161. **The slide-rest** was invented for the purposes of ornamental turning, and is an invaluable adjunct of a good lathe. The diagram is from Mr. Shelley's book and gives a fair idea of its construction, which exhibits the practical value of truth of surface. The object of the contrivance is to give a traversing or sliding motion in two determinate directions to the cutting tool, without any shake or irregularity. For this purpose we require two pairs of true plane surfaces resting upon each other and moved by screws, and accordingly this idea is carried out in the apparatus.

A base plate, *a*, is clamped upon the bed of the lathe, *b*, *b*, and upon it rests a sliding-plate, *c*. The surfaces in contact are true planes, and the joints are dovetailed, whereby the sliding-plate can only be moved in one definite direction at right angles to the bed of the lathe, and this motion is ensured with delicacy and accuracy by the use of a screw turned by a handle.



In like manner, the upper slide, *e*, is traversed by a screw in a plane parallel to the bed of the lathe, and may move in a line at right angles to the direction of motion of the lower slide, or, if preferred, at any angle thereto. It will be seen that the upper slide is attached to a central pin, and that bolts, working in circular slots, enable the workman to fix the direction of motion at any angle he pleases.

Finally, there is the tool-holder, *f*, which holds the tool steadily in any position required for the work done.

CHAPTER XI.

ELEMENTARY MECHANICS OF LIQUIDS AND GASES.

162. We have, in the first chapter, pointed out generally the differences between a solid and a fluid, and purpose now to treat of the mechanical properties of fluids.

Definition.—A fluid is a body which can be divided in any direction, and the parts of which can be moved among one another by any assignable force.

As already stated, there are two classes of fluids, viz., *liquids* and *gases*.

Under ordinary conditions there are many gases which cannot be shown in a liquid form, and there are others which can be readily exhibited as liquids. Hence gases were further subdivided into permanent gases and vapours, the term vapour being applied to a gas which is readily converted into a liquid. Of the *first*, hydrogen and oxygen were examples, as they resisted all attempts made to liquefy them under the action of extreme pressure and intense artificial cold. Of the *second*, steam was a familiar instance; so was the vapour of alcohol or ether; so also was carbonic acid, which was commonly known as a gas, but was not a permanent gas, inasmuch as it could be readily liquefied.

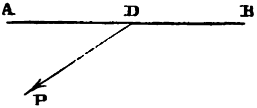
Experiments made within the last few years have shown that the distinction between permanent gases and vapours no longer exists, for even hydrogen has been exhibited as moisture upon glass, and atmospheric air has been liquefied and collected in some quantity at the bottom of a tube, just as if it were water.

163. **Fluid pressure.**—The pressure which a liquid or a gas exerts upon surfaces which confine or sustain it, is a subject for frequent inquiry in mechanics, and we make this our starting-point.

It is necessary to remark that no substance in nature exactly fulfils the definition of a fluid given above. It has been proved that if water be stirred up or agitated in a closed vessel, the friction of the particles of the liquid is competent to raise the temperature of the water. But for all ordinary facts of observation, such as the young student will meet with, the definition may be treated as correct.

164. *Proposition.*—The surface of a liquid at rest is perpendicular to the force which acts upon it at every point.

This follows from the mobility of the particles of a liquid; for let A B represent the surface of a liquid at rest, and P a force acting on a particle D on the surface in direction D P. Then P may be resolved along the surface and perpendicular to it; and, admitting that the particles of the liquid are perfectly smooth, there would be nothing to counteract the resolved part of P which acts along the surface. As long as this component exists there cannot be equilibrium.



Hence equilibrium is only possible when P acts perpendicularly to the surface, in which case the pressure on D may be counteracted by the group of liquid particles surrounding it.

It is on account of the inability of the liquid to oppose any resistance to motion along A B that the bubble of a spirit level moves so readily upon the least change in the inclination of the tube. If gravity be the force acting, as in the case of mercury or water poured into a tray, the surface of the liquid will be a horizontal plane, for gravity acts in a vertical direction.

Accordingly a flat dish with clean mercury in it forms an invaluable true horizontal plane wherewith to view stars by reflected light for astronomical observations. In like manner we can see the image of the moon reflected by water on a very still night.

In all cases where forces come into play, the principle that action and reaction are equal and opposite will apply.

165. A fluid acted on by force presents a surface which is everywhere perpendicular to the action of the force; and conversely, a surface supporting a fluid which presses upon it will supply a reaction everywhere perpendicular to the surface. Whether the aggregate of the pressures produces the surface, or the surface gives rise to the pressure, is immaterial.

Hence we conclude, that in all cases of fluid pressure upon a surface, whether due to the action of a liquid or a gas, the pressure upon the surface acts perpendicularly thereto at every point of it.

This fact is fully recognised by engineers in all practical applications.

166. In like manner, the pressure on the surface separating any one liquid particle from its neighbour is perpendicular to that surface. Nothing is known about the surfaces which bound a liquid particle, but it is assumed that the directions in which these surfaces can lie are infinite in number, and that any one of them may determine the direction in which any given pressure is being transmitted.

Hence we infer that a fluid transmits pressure in every direction at once through chains of liquid particles; and, since there is no sensible friction between the particles whereby to absorb the pressure, there is no loss in the transmission.

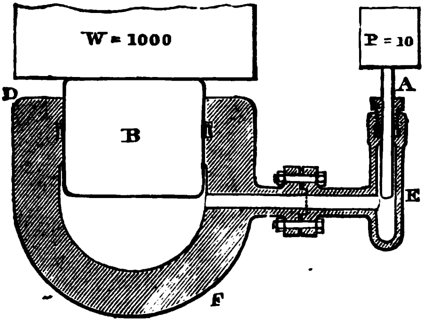
167. **Transmission of fluid pressure.**—Fluids transmit pressure equally in all directions, and there is no loss of pressure in the transmission. This is true for both liquids and gases. Here, then, is a fundamental point of distinction between a solid and a fluid.

When pressure acts on a solid, it is felt only in the line of its direction.

But when pressure is exerted at any point of a liquid or gas, it is felt not only in the line of its action, but at every other point of the interior of the fluid at the same instant, and in every possible direction.

The molecules of a liquid can only support each other by direct pressure; they do not cohere, they are not rough like the particles of sand, they are ever ready to slide upon each other where any pathway is open, and they press outwards in any direction in which motion is possible.

168. The following drawing from Sir J. Anderson's series shows the principle of the transmission of fluid pressure as applied in the hydrostatic press. Hereafter we shall give a drawing to show the valves and the method of working the press; at present we exhibit only the principle of its action.



A solid plunger, A, loaded with a weight of 10 lbs., passes through a watertight collar into a vessel, E, containing water. The vessel, E, communicates by a pipe with another strong vessel, D F, into which is fitted a ram, B, passing through a watertight collar and sustaining a load, W.

The downward pressure of the plunger, A, on the water in the vessel, E, is transmitted at once through the whole liquid mass. There is an upward pressure on each square inch of area of the ram, B, which is exactly equal to the downward pressure exerted on the water by each square inch of the area of the base of the plunger, A.

Let the area of the base of the plunger, A, be 1 square inch, and that of the base of the ram B be 100 square inches.

Then pressure on B $= 100 \times$ pressure on A.

Hence, if pressure on A $= 10$ lbs.,

$$\text{pressure on B} = 100 \times 10 = 1{,}000 \text{ lbs.}$$

This is the principle of the hydrostatic press; the power of the press increases in the proportion in which the area of B is greater than the area of A.

The principle of work applies equally in this case.

Thus, let the plunger, A, descend 5 inches, then the volume of water forced into D F

$$= 5 \times 1 = 5 \text{ cubic inches.}$$

Also let B rise through a height x, then area of B $= 100$ square inches.

$$\therefore 100x = \text{volume of water forced into D F.}$$
$$= 5 \text{ cubic inches.}$$
$$\therefore x = \tfrac{5}{100} = \tfrac{1}{20}.$$

Hence motion of B $= \frac{1}{100} \times$ motion of A;

or if motion of B $= 1$, motion of A $= 100$.

By the principle of work, if 10 lbs. on A supports W lbs. on B, we have

$$\text{W} \times \text{motion of B} = 10 \times \text{motion of A},$$
$$\text{or W} \times 1 = 10 \times 100,$$
$$\text{or W} = 1{,}000 \text{ lbs. as before.}$$

169. **Definitions.**—The pressure in the interior of a fluid may be *uniform* or *variable.*

When *uniform*, it is the pressure on a square inch of surface exposed to the fluid pressure.

When *variable*, it is the pressure which would be exerted on a square inch of surface, if the pressure were the same throughout and identical with that at the point considered.

We write *p* for the pressure of a fluid at any point of a surface, and *p* means the pressure per square inch, unless the contrary be expressed. This is true alike for liquids and gases. It is the practice to speak of the pressure of steam as being (say) 50 lbs., meaning thereby 50 lbs. per square inch on the shell of a boiler.

170. The mass of a fluid is estimated in the same way as the mass of a solid.

Definition.—The *density* of a fluid is the number of units of mass contained in a unit of volume. A cubic foot may be taken as the unit of volume, and a pound avoirdupois as the unit of mass.

One cubic foot of water weighs 62·5 lbs., hence the density of water is 62½ lbs. to the cubic foot. More accurately, a cubic foot of water at 39·4 F. weighs 62·425 lbs.

In round numbers, it is usual to say that a cubic foot of water weighs 1,000 ounces, and we see that (the number of ounces in a pound avoirdupois being 16):

$$\frac{1,000}{16} = \frac{125}{2} = 62{\cdot}5.$$

The weight of a cubic yard of water is a little less than ¾ of a ton.

171. That air, as well as water, has weight is proved by the following experiment:

A glass globe E, provided with a stopcock, is screwed on to the plate of an air-pump, and the air is exhausted. The globe is then attached to the scale pan of a balance and carefully weighed.

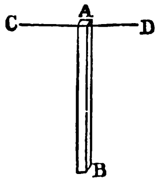
The globe is next screwed to the neck of a graduated jar, with an open base, filled with air, and placed with its open base in a tray of water. By this arrangement the air in the jar is cut off from the external atmosphere. The stopcock is again opened, when air rushes into the globe, and the water rises in the jar, and indicates by its altered level the quantity of air which has entered the globe.

The globe is now weighed, and is found to be heavier by several grains. This increase is due to the weight of the measured quantity of air which has entered the globe.

When the experiment is very carefully conducted, it appears that 100 cubic inches of air at 60° F. weigh 31 grains very nearly, the barometer standing at 30 inches. The student will understand the part played by the barometer when he has read the account of that instrument.

172. **Pressure of liquids.**—In a liquid at rest under the action of gravity, the pressure varies as the depth below the surface.

Let C A D be the surface of a liquid at rest, and conceive that the portion contained within a slender prism, A B, having vertical sides and a horizontal base, is converted into a solid. This will neither alter the pressure nor disturb the equilibrium of the surrounding fluid.



Now the horizontal forces are separately in equilibrium, and so are the vertical forces. The only vertical forces acting are the weight of the prism and the upward pressure on the end B, for we disregard or neglect any pressure at A. Hence

pressure on end B = weight of prism.

Let p = pressure at B, a = area of base of prism.

A B = z, weight of cubic inch of liquid = w.

Then $$p\,a = a\,w\,z$$

or $p = w\,z$, or pressure varies as depth.

For small depths we may take inches as units, for greater depths we measure in feet or fathoms.

It will be proved hereafter that the atmosphere presses on A with a considerable force, but inasmuch as we are not sensible of this pressure under ordinary circumstances, it is usual to disregard it.

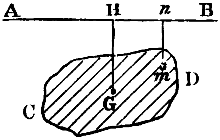
If the pressure of the atmosphere were taken into account, we should add that pressure to p, and the law would be modified accordingly.

The student will now understand the meaning of the expression, '*pressure of a head of water.*' When there is a reservoir of water above the level of the supply, the water rushes out under pressure according to the above law, and the term '*head*' is a short way of conveying an idea of a difference of vertical height, whereby, if we say there is a head of 20 feet, we mean that the point from which the water is drawn off is 20 feet below the level in the reservoir.

Proposition: The pressure of a liquid on any area is equal to the weight of a column of the liquid whose base is the area pressed, and height is the depth of the centre of gravity of the area below the surface of the liquid.

In order to prove this we begin with an easy case, viz. where the liquid is contained in a vessel with plane vertical sides, the area pressed being a portion of one of the sides.

Let the surface of the liquid meet the side in question in A B, also let C D be the given area, and G its centre of gravity. Draw G H perpendicular to A B. Let m be a small piece of the surface at depth $m\,n$ or x, and let m be expressed in square feet, and x in feet.



Then pressure on $m = m\,x \times$ weight of a cubic foot of the liquid, and the like for all portions similar to m.

$\therefore$ press. on surface C D = (sum of all portions such as $m\,x$) $\times$ weight of cubic foot of the liquid.

But by a property of the centre of gravity, area C D $\times$ G H = sum of all portions such as $m\,x$.

$\therefore$ press. on surface C D = area C D $\times$ G H $\times$ weight of cubic foot of liquid.

Which proves the proposition.

Ex.: A tank, in the form of a cubical box, whose sides are vertical, holds four tons of water when quite full. What is the pressure on its base, and what is the pressure on one of its sides? (Science Exam. 1880.)

The sides of a cubical box being all equal, the pressure on the base = weight of water in tank = 4 tons.

Also, area of vertical side = area of base.

But depth of centre of gravity of vertical side = $\frac{1}{2}$ depth of centre of gravity of base.

∴ press. on vertical side = 2 tons.

Ex. : A water tank 6 feet long and 6 feet wide, with an inclined base, is 6 feet deep at the front and 3 feet deep at the back, and is filled with water. Find the pressure on each of the four sides and on the base; water weighing $62\frac{1}{2}$ lbs. per cubic foot. (Science Exam. 1882.)

Here the depths of the centres of gravity of the respective sides are 3, $1\frac{1}{2}$, $\frac{7}{3}$, and of the base $4\frac{1}{2}$; the corresponding areas being 6×6, 6×3, $6\times 4\frac{1}{2}$, $6\times 6{\cdot}7082$. Whence the required pressures are 6750 lbs., 1687·5 lbs., 3937·5 lbs., and 11320·1 lbs.

Ex. : To find the pressure on the internal surface of a hollow sphere when filled with water.

Let a = radius of sphere, w = weight of a cubic inch of water.

Then press. on interior surface = area of surface × depth of centre of gravity × w.

$$= 4\,\pi a^2 \times a \times w.$$

Also weight of water in sphere $= \dfrac{4\,\pi\, a^3 w}{3}$.

∴ press. on interior surface = 3 × weight of water.

Conceive the sphere divided into two parts by a vertical partition through the centre, and that only one compartment is filled with water.

Then press. on curved surface = $2\,\pi a^3 w$.

press. on partition $= \pi a^2 \times a \times w = \pi a^3 w$.

∴ press. on curved surface : press. on partition :: 1 : 2.

173. This last example leads us to distinguish between the total pressure on a curved surface and that portion of it which is perpendicular to any given plane. We have

here a case in which one pressure is exactly double the other pressure.

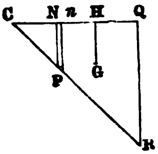
Another point of great importance is the following :—

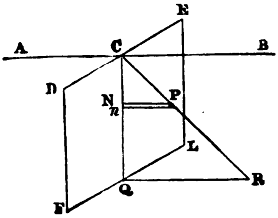
The pressure on the base of a vessel containing any liquid is independent of the form of the vessel. This is a direct consequence of the fact that the pressure on the base depends only on the area of the base, the depth of its centre of gravity below the surface of the liquid, and the density of the liquid.

In the case supposed not one of these three things can admit of any change whatever, and therefore the pressure remains constant.

174. **The centre of pressure** of a plane area immersed in a liquid is that point in which the resultant pressure of the liquid acts.

It has just been shown that the pressure of a liquid upon a surface increases in direct proportion to the depth ; that is to say, the pressure at a depth of 4 feet is twice as great as at a depth of 2 feet, and so on. We may conceive, therefore, that a vertical strip of surface exposed to the pressure of a liquid—such, for instance, as a vertical beam of timber forming part of a lock-gate—is exposed to a continually increasing pressure for each foot of its depth, according to the law proved in the last article.





In order to obtain some definite knowledge of the subject, the student may examine the case of a rectangle, D L, having the side, D E, in the surface of a liquid, and so placed that the plane, D E L F, is vertical. Let C Q be the edge of a narrow vertical strip made by two vertical planes very close together.

Draw QR horizontal and equal to CQ; join CR; also from any point, P in CR, draw PN parallel to RQ, and complete the rectangle P*n*, whose vertical depth, N*n*, is very small.

Since the pressure varies as the depth, the pressure at Q is proportional to CQ, and therefore to QR, so likewise the pressure at N is proportional to NP. It becomes evident, therefore, that the pressure on N*n* is represented very approximately by the area P*n*, and the like would be true for other small portions into which CQ may be divided, wherefore the whole horizontal pressure of the liquid on CQ may be represented graphically by the area of the triangle CQR.

In other words, if CQ were a horizontal line, and the triangle CQR were made up of a series of heavy wires hanging upon CQ, we should create a mechanical representation of the actual fluid pressure upon CQ.

Let G be the centre of gravity of CQR, draw GH vertical, then CH = $\frac{2}{3}$ CQ.

But inasmuch as the resultant pressure of all the heavy wires hanging on CQ acts through H, so also, when CQ is subjected to fluid pressure, the resultant of all the separate increasing pressures acts also through H; that is, the depth of the centre of pressure is $\frac{2}{3}$ the depth of the immersed edge, DF.

It is also evident that the centre of pressure lies in a vertical line bisecting DE.

175. **Law of Archimedes.**—The power of floating in liquids, or even in gases, which many substances possess, is full of interest, because we recognise in it the evidence of a natural law. The discovery of the law is due to Archimedes, and in proving it we shall assume that when a liquid is at rest any given portion of it may be solidified without a change of volume.

This is a physical impossibility. Water increases in bulk when it changes into ice: that is the reason why ice floats upon water, and accounts for the fact that water pipes often burst during a frost. Molten wax diminishes in bulk as it solidifies, as may be seen by pouring molten wax into a jar and allowing it to cool; and as a rule there is some change of volume when any liquid solidifies.

But the solidifying which we assume is imaginary, not real, and we argue thus: The liquid is at rest, none of its particles are moving, and it cannot alter the pressure or disturb the equilibrium of the surrounding fluid if we suppose a certain number of particles to be deprived of the power of motion, or to be connected rigidly together, that is, to be solidified.

The complete statement of the conditions of equilibrium of a floating body is the following:

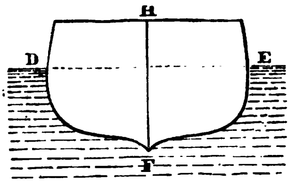
1. When a solid floats in water, the weight of the solid is equal to the weight of the water displaced.

2. The straight line which joins the centres of gravity of the solid and water displaced is vertical.

In order to establish the law, let us suppose that a portion of water, as DEF, contained within a mass of water at rest, becomes solidified. This will not alter the pressure of equilibrium of the surrounding water. Then DEF is a solid at rest under:

(1.) Its own weight acting downwards through the centre of gravity.

(2.) The pressure of the surrounding water.



These are the only forces acting, and the body DEF is at rest; therefore these forces are equal and opposite, and act in the same vertical line.

Conceive now that a ship, HF, floating in the water, displaces the same volume, DEF. The pressure of the water will be the same as before, and act in the same line as before, while the weight of the ship must act in a vertical line through the centre of gravity of the ship.

If there be equilibrium, these two forces must be equal, and must act in the same straight line in opposite directions.

Therefore the weight of the ship or solid must equal the weight of the water displaced, and the line joining the centres of gravity of the solid and water displaced must be vertical.

Whether the solid be wholly or partially immersed in the liquid, the application of the law is the same. The weight lost is always equal to the weight of the liquid displaced.

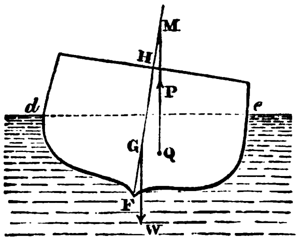
Since air has weight, we infer that the law of Archimedes will apply equally in the case of gases. That the fact is so will appear directly from the following experiment. A hollow copper sphere is suspended from one arm of a balance, and is counterpoised by a small leaden weight on the other arm, the beam of the balance being horizontal. The apparatus is then placed under the receiver of an air-pump, and the air in the receiver is pumped out; when it appears that the copper ball goes down, and is no longer balanced by the lead weight.

Suppose the volume of the ball to be 10 cubic inches, when the weight of the air which it displaces will be 3·1 grains. Add this to the leaden weight, and it will be found that the ball is overbalanced in the air, but exactly counterpoised when tested under the exhausted receiver.

The student will now comprehend the reason why a light silk balloon will carry up a car with people in it. The balloon is inflated with a gas lighter than air, and air has weight, as we have shown. Taking larger units, we may say that 13 cubic feet of air weigh 1 lb. The volume of a balloon is considerable in order that it may be effective; thus, the volume of a balloon competent to carry up three persons would probably be some 700 cubic yards, or 18,900 cubic feet. Of course the buoyancy in this case will be the difference of weight of the enclosed gas and the displaced air.

176. **The meta-centre.**—Returning now to the vessel floating in water, we have to determine whether its equilibrium is stable or unstable. It will appear that the nature of the equilibrium depends on the position of a certain point, technically known as the *meta-centre.*

The drawing shows a section of the vessel H F when heeling over to a very small amount. Let G be the centre of gravity of the vessel, Q that of the water displaced, then the weight of the vessel is a vertical force, W, acting through G, while the upward pressure of the displaced water is a vertical force, P, acting upwards through Q.



These forces balance, for otherwise the vessel would rise or sink, and being equal, they produce a couple whose arm is the perpendicular distance between G W and Q P.

Let Q P meet F G H in M, then the point M is the meta-centre, and is a point of much importance, though not readily found, as the calculation of its position demands a knowledge of the calculus.

The student will perceive that, so long as M lies *above* G, the tendency of the water pressure will be to restore F H to the vertical position, whereas, if M lies *below* G, the two forces in action will tend to deflect F H still more, and the vessel will fall over.

In order that a vessel may float securely, the point G must lie below M. If G ever rises above M, the vessel will assuredly fall over on its side in the water. This may explain an accident which has sometimes happened where the cargo is taken out of a vessel without putting in ballast at the same time. The weight of the masts and rigging may cause G to rise above M.

Hence also the risk of upsetting a small boat when people stand up in it.

177. **The specific gravity,** or specific density of any *solid* or *liquid*, is the ratio which its density bears to that of distilled water at 60° F.

It is on account of the expansion of bodies by heat that a fixed temperature 60° F. is chosen as the common temperature of the solid, liquid, and water.

Tables of specific gravities are formed for reference, of which a specimen is appended. Thus the specific gravity of mercury is 13·5, by which we intend that any given bulk of mercury will weigh 13·5 times as much as an equal bulk of water at the same temperature.

The natural law that when a body is immersed in a liquid, it loses as much weight as the weight of the liquid displaced, is the foundation of the method adopted for finding the specific gravity of solids.

The apparatus required is a good balance. The solid must sink in water, and must not dissolve like sugar or salt, and we then attach it by a horse-hair to one pan of the balance, and weigh it (1) in air; (2) when hanging in water. For example:

Let a piece of lead weigh in air . :	820 grs.
" " " water .	749 grs.
Loss, being weight of water displaced .	71 grs.

Hence specific gravity of lead $= \frac{820}{71} = 11{\cdot}55$, or, to put the method in a general form,

Let w = weight of body in air,
" x = " " water.

Then weight of body − weight of water displaced = x,

$$\therefore \text{ weight of water displaced} = w - x,$$

$$\therefore \text{ sp. gr. of body} = \frac{w}{w-x}.$$

If the substance be soluble in water, it may be weighed in [illegible] not [illegible] son- [illegible] to a [illegible] ned.

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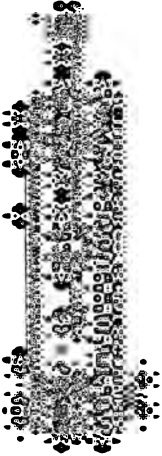

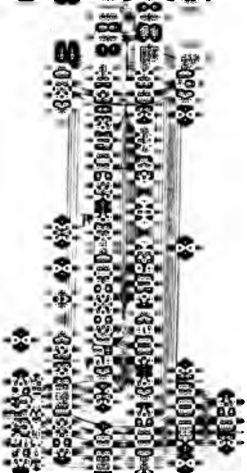

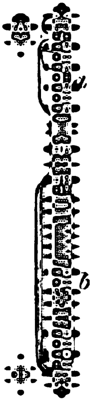

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(1) *Nicholson's* hydrometer consists of a hollow cylinder, B, which ensures floatation, having at its base a loaded pan, C, to keep it upright, and at the top a stem supporting a dish, A. Upon the stem a standard point, *o*, is marked.

This instrument may be used for finding the specific gravity of a solid or liquid.

As for the solid, let that be (say) a piece of sulphur. Put the hydrometer in water, when it will require a given weight placed in A in order to sink the instrument to *o*. Let this weight be 125 grains. Now place the sulphur in A, and add (say) 55 grains in order to sink the instrument again to *o*. It follows that the weight of the sulphur is 70 grains. Next place the sulphur in C, as marked *m*, and let 34·5 grains be placed on the dish A, in addition to the 55 grains, in order again to sink the instrument to *o*.

$$\text{Then} \qquad \text{weight of sulphur} = 70 \text{ grains.}$$

$$\text{weight of water displaced by sulphur} = 34{\cdot}5 \text{ grains.}$$

$$\therefore \text{ specific gravity of sulphur} = \frac{70}{34{\cdot}5} = 2{\cdot}03.$$

In order to find the specific gravity of a liquid (B). Let x be the weight which sinks the instrument to the point *o* in water, y the weight which sinks it to the same point in the liquid (B), and let W be the weight of the instrument.

$$\text{Then weight of water displaced by instrument} = W + x.$$

$$\text{,, \quad liquid (B) \quad ,, \quad ,,} \quad = W + y.$$

$$\therefore \text{ specific gravity of liquid B} = \frac{W + y}{W + x}.$$

(2) *Tweddel's* hydrometer is used for finding the specific gravity of liquids heavier than water, such as sulphuric acid, and it sinks to different depths as the density of the liquid varies, a greater portion of the stem being immersed when the liquid under trial is of less specific gravity, and the range being determined beforehand when the instrument is designed. It therefore acts by *variable immersion*, the load remaining constant.

There are other hydrometers constructed on the principle of variable immersion, such as those for determining the density of alcohol, which is lighter than water, and those employed for determining the density of salt water in a boiler, where the graduations are so arranged as to indicate in a ready manner either the strength of the alcohol or the quantity of salt held in solution in the water.

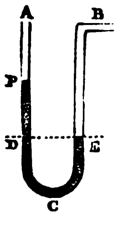
Tweddel's hydrometer is made of glass, with two globes, one for floatation, the other for keeping it upright, and the stem is so graduated that the reading of the number of degrees, multiplied by 5 and added to 1,000, gives the specific gravity of the liquid as compared with water, whose specific gravity is 1,000. Thus 15 Tweddel represents the specific gravity of 1,075, or, calling the specific gravity of water 1, it represents a specific gravity 1·075.

(3) The specific-gravity bottle, in one form, is a bottle, *b*, with a fine stem, *c*, ending in a wide tube, *a*, having a glass stopper.

The bottle is first weighed when empty, then when filled with water up to the mark *c*, and, finally, when filled with a given liquid up to the same mark.

We thus ascertain the weight, (1) of a given volume of water, (2) of the same volume of a given liquid, and the specific gravity of the liquid becomes known.

179. **Pressure gauge.**—One form of this instrument is shown in the sketch. It consists of a bent tube, A C B, with parallel legs, and open at both ends. A small quantity of liquid is poured into the tube, so that the leg, B C, becomes nearly half full. This liquid would be water, when small pressures (such as that of gas for the supply of houses) were to be measured. Otherwise it would be mercury.



When in use, the end B is attached by an air-tight joint to the gas or air under pressure, the end A remaining open, and the legs standing in a vertical position.

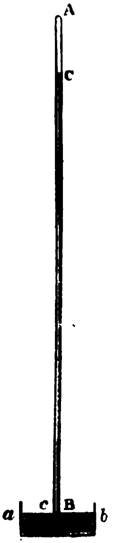
The liquid will then rise in A C and sink in B C, and the difference of level in the two tubes will give the required excess of pressure above that of the atmosphere.

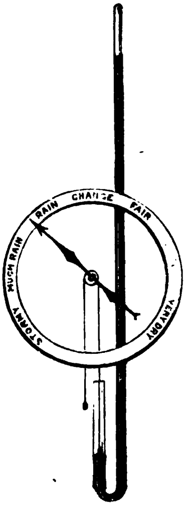
Let D E be a horizontal plane touching the surface of the liquid in B C, and P D the difference of level in the two legs. Then pressure at E = pressure at D, because D E is horizontal,

and press. at D varies as P D
∴ press. at E „ „ P D.

By attaching a scale to P D, and remembering that one inch of rise at P corresponds to two inches in difference of level, it is easy to arrange that the scale shall give the required pressure in a convenient form.

180. **The barometer** is used for measuring the pressure of the atmosphere, and was invented by Torricelli in the year 1643.





A tube, A B, some 34 inches long, closed at one end and about one-third of an inch in diameter, is filled completely with clean mercury, and inverted in a small vessel of the same liquid. The mercury will now sink in the tube, leaving an empty space, A C, but a column, C B, about 30 inches long, will remain supported by the pressure of the atmosphere, and will give, by its length, an accurate measure of the air pressure at the level *a b*.

For simplicity, let the internal area of a section of the tube be 1 square inch, then the pressure per square inch inside the tube at the level *a b* is the weight of the column B C. But this pressure is the same as that of the air outside on *a b*, hence

press. of atmosphere on 1 sq. inch = weight of column B C.

Let B C = h, w = weight of cubic inch of mercury.

$\therefore$ press. of atmosphere = $w h$.

181. An ordinary barometer is constructed on the principle described, and is merely a glass tube filled with clean mercury by the process of boiling, the object being to exclude all traces of air and moisture from the empty space at the top of the tube. The tube is inverted with its open end in a cistern of mercury.

Or the cistern may be dispensed with, and the barometer may be a simple bent tube, as in the second diagram. There should be a carefully graduated scale, attached to the upper end of the tube, showing the exact height of the column. In barometers of the old-fashioned type the level has been shown by a cord passing over a pulley and moving an index-finger, as in the sketch.

182. The standard height is sometimes taken as 29·922 inches of mercury, in which case the pressure of the atmosphere is 14·7 lbs. on the square inch, or 2116·4 lbs. on the square foot.

Taking the specific gravity of mercury as 13·5, and the mercurial barometer standing at 30 inches, a water barometer would indicate 30 × 13·5 inches = 405 inches = 33¾ feet. The water barometer was constructed soon after the mercurial barometer, in order to test the theory propounded by Torricelli.

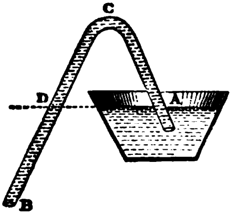
183. **The siphon** is an instrument much used for drawing liquids out of casks. It is then fitted with a tube for sucking out the air, and has a stopcock or tap at the end. The sketch merely shows a bent tube, A C B, open at both ends, and dipping into a vessel of the same liquid, which is all that is required in order to exhibit the action of the siphon.

The tube A C B is first filled with liquid, which is prevented from running out, and the tube is then inverted with one end, A, in a vessel of the same liquid, the end B being at a lower level than the surface of the liquid in the vessel.

Let the plane of the liquid surface cut the tube in D, then

pressure within the tube at D = pressure at A
= pressure of atmosphere.

But the pressure within the tube of B is greater than at D, and is therefore greater than the pressure of the atmosphere. It follows that the pressure of the liquid at B is sufficient to overcome that of the air from without, whereby the column B C tends to separate at C, and to run out at B.

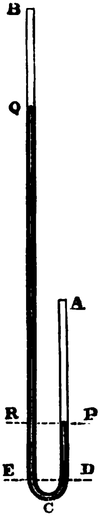


If the height of C above A D be less than that of the liquid in a barometer-tube, the pressure of the air will prevent any separation at C, and will keep up a continuous stream by forcing the liquid to ascend A C. This will continue so long as B is below the level of the plane A D.

184. **Boyle's law** was discovered in England in 1662 by Robert Boyle, and was discovered independently in France in 1676 by Marriotte. It is referred to by many writers as Marriotte's law, and may be thus stated :—

The pressure of a gas at a given temperature varies inversely as its volume. That is to say, if the pressure of air contained in a vessel *one* cubic foot in bulk be 8 lbs. on the square inch, the pressure will be 16 lbs. per square inch when the volume is reduced to ½ cubic foot, and 2 lbs. per square inch when the volume is increased to 4 cubic feet.

This is an experimental law, and is proved as follows: Take a glass siphon tube, A C B, some 40 inches in length, with parallel legs, and either sealed or fitted with a stopcock at A. Having placed it with the axis of either tube vertical, pour a small quantity of mercury into BC, and, by opening the stopcock or withdrawing some air from AC, make the mercury stand at the same level, ED, in both branches.



The air in AD being now completely separated from that in BC, pour in more mercury *slowly*, and the level in the two legs will take the positions Q and PR, where PR is a horizontal plane touching the surface in AC. Let h be the height of mercury in a barometer at the time of the experiment.

Then $$\frac{\text{pressure at R}}{\text{pressure at Q}} = \frac{h + \text{RQ}}{h}.$$

But pressure at R = pressure of air in AP,
pressure at Q = pressure of air in AD.

Therefore $$\frac{\text{press. of air in AP}}{\text{press. of air in AD}} = \frac{h + \text{RQ}}{h}.$$

The positions of D and P being carefully marked on the tube, the volumes AP, AD must be ascertained, which may be done by weighing the quantities of mercury they will contain, when it will appear that,

$$\frac{\text{volume AD}}{\text{volume AP}} = \frac{h + \text{RQ}}{h}.$$

Hence $\dfrac{\text{pressure of air in AP}}{\text{pressure of air in AD}} = \dfrac{\text{volume AD}}{\text{volume AP}}$, which was to be proved.

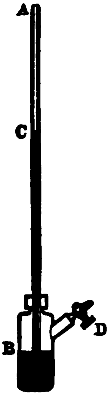
To express the law in symbols, let volume AP $= v$, pressure in AP $= p$, then the volume AD is constant, and so is the pressure in AD.

$$\therefore\ p = \frac{\text{constant}}{v}, \text{ or } p\,v = \text{constant} = \text{C}.$$

Many years ago, Boyle's law was tested for air up to a pressure of 27 atmospheres, and was believed to hold strictly, but later experiments have shown that, under increased pressure, the volume of air contracts a little more than accords with Boyle's law. Again, hydrogen does not yield to great pressures quite so much as if the law maintained strictly, and carbonic acid exhibits deviations even under moderate pressures.

For all ordinary purposes, however, the student may assume that the law is strictly true, for the deviations are inconsiderable except when the gas approaches the state of liquefaction.

185. **A manometer** is an instrument employed for measuring the pressure of compressed air or other gas, and may assume a variety of forms. One illustration is given in the diagram. Here the tube, A B, is a barometer tube closed at A, and dipping into some mercury in a bottle to which a stopcock, D, is fitted. When D is opened let the mercury stand at the same height both within and outside the tube. When air under pressure is allowed to enter the bottle through the passage D some mercury will be forced up the tube, the air in A C will be compressed according to Boyle's law, and the exact pressure may be determined by observing the position of the point C. The calculation for this purpose is rather beyond the scope of this book.

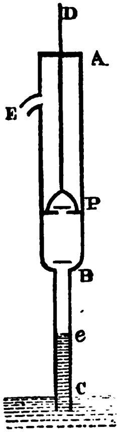


If the tube A B be open at the top and be several feet in length, the apparatus forms a convenient pressure-gauge; for if the barometer stand at 30 inches, indicating (say) a pressure of 15 lbs. on the square inch, and the height of the column of mercury, C B, is 8 feet, the pressure of the air in the bottle will be $\frac{12 \times 8 \times 15}{30} =$ 48 lbs. per square inch. Pressure-gauges for steam boilers are tested by comparison with an apparatus of the kind described.

186. **The suction-pump** is said to have been invented 200 years before the Christian era, but its action was first explained by Torricelli, who, as we have seen, invented the barometer.

If air be exhausted or sucked out in some manner from the top of a pipe dipping into water, the pressure of the air will force some water up the pipe, and is competent to sustain a column about 32 feet in height. If part of the water so raised be removed, more will be pressed up to supply its place, and this is the action of the suction-pump, which is dependent on the pressure of the atmosphere.

Let A B, B C be two hollow cylinders, fitted with a valve opening upwards, and worked by a rod, P D. There is also a valve at B opening upwards, and the tube B C dips into a cistern of water at a depth less than the height of a column of water in a water barometer.



Suppose P to be at B, no water having risen up B C. Let the piston be raised, then the valve P will remain closed, but the pressure in P B becomes less, whereby the valve B opens and the pressure in B C also becomes less. Hence the water will rise in B C to some level, *e*. On the descent of the piston, P opens, B closes, and the air in P B escapes through P, the column C *e* remaining stationary.

On the next ascent of the piston the action before described is renewed, until, finally, the water rises through B. It is then compelled to pass through the valve P, and is lifted by the piston till it escapes at the spout E.

Neglecting friction, the work done in lifting the pump bucket depends on the difference of pressure of the air on the upper and under surfaces thereof.

At first these pressures are equal. When the pumping begins the pressure on the under surface becomes less and the tension of the pump rod increases. In order to ascertain its amount we must find the pressure of the air below the piston, P. The student will remember that the valve B is open when the piston is rising.

Let A be the sectional area of A B in square inches, and let $ce = 12x$ inches, $12h$ being the height of water in a barometer in inches, and w being the weight of a cubic inch of water in pounds.

Then pressure

$$\text{on area A of air below P} + 12x\,\text{A}\,w = \text{press. of atmosphere.}$$
$$= 12h\,\text{A}\,w.$$

$$\text{Also press. on area A of air above P} = 12h\,\text{A}\,w,$$
$$\therefore \text{ tension of pump-rod} = 12x\,\text{A}\,w.$$

This expression may be put in another form by observing that

$$\text{pressure of atmosphere on 1 square inch} = 12hw = 15 \text{ lbs.}$$
$$\text{or } \quad 12w = \frac{15}{h},$$
$$\therefore \text{ tension of pump-rod} = \frac{15x\,\text{A}}{h} \text{ lbs.}$$

The formula shows that the tension of the rod is independent of the form or size of the pipe leading down to the water. It depends on the sectional area of the pump barrel, and on that alone. If B C were a cone with a wide mouth at C, the tension would be just the same, for the pump-rod in effect lifts a column of water whose base is the area of the cylinder A B and height the altitude of the water lifted.

187. **The force-pump,** or plunger-pump, is the same apparatus as that already described, so far as raising or sucking up the water is concerned, but it differs in the mode of sending it to a higher level than the base of the pump. It consists of a plunger, C P, working in a cylinder, D F, and connected with an air vessel, E, having a pipe, H, reaching nearly to its base. There are valves, A and B,

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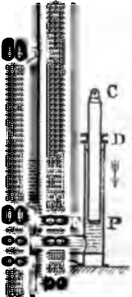

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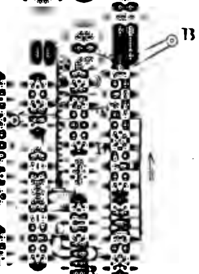

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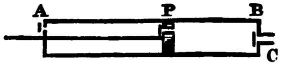
downward pressure of the atmosphere, which would enormously increase the labour of working the pump if it were not counterbalanced. It is an instructive experiment to ascertain by trial the amount of force necessary to draw up a piston 3 or 4 inches in diameter from the bottom of an open cylinder, into which it fits closely, so that the air cannot leak into the space underneath. In an air-pump the pressure of the air above the piston would be 14·7 lbs. on the square inch, and that below the piston would soon fall to less than 1 lb., which numbers show very clearly the necessity for some such provision as the double-cylinder arrangement.

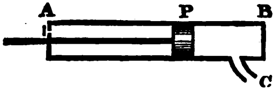
189. **Smeaton's air-pump** is a single-cylinder pump, closed in at one end, as shown, and provided with a valve, A, which relieves the piston from the pressure of the external air. The pipe, C, communicates with the receiver, and the air which gets into the space, A P, is expelled through the valve, A, at the end of each stroke of the piston to the left hand. In other respects the pump is identical with that already described. Since the valve in P is now required to open against the diminished pressure of the air in A P instead of against the full pressure of the atmosphere, the exhaustion obtained is more complete than in Hawksbee's pump.



190. It is now time to call attention to the construction of the valves of an air-pump, for the student will find that, in the attempt to obtain a nearly perfect vacuum, the important steps in invention have been, first, to work with only one valve, and finally, to abolish valves altogether, and to employ falling mercury as an equivalent for the piston and valves.

An air-pump valve is made by stretching a small strip of oiled silk over a hole in a plate. The ends of the strip are tied down by a thread, leaving the sides open. The silk is lifted by the pressure of the air forced through the valve. When the silk is pressed on the plate the valve is perfectly closed. It follows that the air within the channel of the valve must exert sufficient pressure to lift the silk, or it cannot escape, and after a time the exhaustion will stop.

191. **Grove's air-pump** shows the first systematic attempt to get rid of the valves. It is Smeaton's pump with the valves P and B abolished. The pipe, C, which leads to the receiver is now brought to a point intermediate between A and B, whereby the work of pumping is effected by the solid piston, P. The artifice consists in allowing P to pass beyond C. The air in the receiver then expands, and fills the space between A and the piston. On the return stroke the air enclosed in A P is swept out of the valve A. It is clear that the exhaustion will go on until the air compressed in the passage through the plate, A, becomes unable to lift the valve A. The principle is an excellent one, and it only remains to get rid of the valve A.



In order to carry out this idea completely, the cylinder, A B, is a glass tube some five feet long, having a passage near the top leading to the receiver, which is to be exhausted. The end, A, of the tube dips into a cistern of mercury, and drops of mercury, falling in quick succession from a vessel connected with B, sweep the air out of the tube, leaving the receiver itself in the same condition as the empty space at the top of a barometer tube. This is the principle of *Sprengel's* air-pump.

192. To find the density of the air in the receiver of a pump after any number of strokes.

Let A be the volume of the receiver,

B ,, ,, cylinder or barrel.

$d_2 \cdot\cdot d_n$ the densities of the air in the receiver after

1, 2 ·· n strokes of the piston, and let d be the density of the atmosphere.

Then
$$d_1\,(\mathrm{A}+\mathrm{B}) = d\,\mathrm{A},$$
$$d_2\,(\mathrm{A}+\mathrm{B}) = d_1\,\mathrm{A}, \text{ and so on,}$$
$$\therefore\quad d_1 d_2\,(\mathrm{A}+\mathrm{B})^2 = d\,d_1\,\mathrm{A}^2,$$
$$\text{or}\quad d_2 = \frac{d \times \mathrm{A}^2}{(\mathrm{A}+\mathrm{B})^2}.$$
$$\text{In like manner } d_n = \frac{d \times \mathrm{A}^n}{(\mathrm{A}+\mathrm{B})^n}.$$

193. Referring to Art. 180 for the drawing of a barometer tube, the student will comprehend the so-called barometer gauge for an air-pump. Conceive that the tube A B dips into mercury at B, and is connected at A with the receiver of an air-pump. It is clear that as the pumping goes on the mercury will rise in A B, and the height to which it rises will furnish a measure of the degree of exhaustion of the air. Thus, if h be the altitude of mercury in a barometer, z its altitude in the tube A B, d the density of the atmosphere, we have

$$\text{density of air in receiver} = \frac{h-z}{h} \times d.$$

Again, by referring to the drawing in Art. 179, the student will understand the siphon gauge of an air-pump. Here A C B is a siphon tube, A C being about 4 inches long and closed at the end A, the end B being connected with the receiver.

The whole of the tube A C is filled with mercury, the tube C B being empty, and the surface of division between the mercury in A C and the air in C B is in the bend at C. When the air in the receiver is sufficiently exhausted the mercury will fall in A C and rise in C B, as in the diagram, and the difference of level in C A and C B will measure the degree of exhaustion.

194. **The condensing syringe** consists of a cylinder, A B, open to the air at A, and having two valves, one in the

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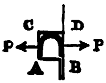
[illegible] f the [illegible]; this [illegible] r-tight [illegible] d the [illegible] chine. [illegible] siderable, and often exceeds two tons on the square inch. Two forms of packing are commonly adopted—(1) *the ring*, (2) *the cup leather*.

A section of the ring is shown as enclosed in the chamber A B C D, D B being the side of the ram. The water under pressure leaks into the chamber and forces the leather tightly against the sides. The greater the pressure the more difficult it is for the water to pass beyond the ring. The *cup leather* is a simple cup fastened to one end of a piston, which acts only when there is water pressure on the inside of the cup.



The press is shaped as in Art. 166. The hemispherical base is important in large presses, the metal being sounder and less liable to yield than if the base were flat. The force-pump and valves will be understood from the drawing.

Let $r =$ radius of ram, $s =$ radius of plunger, P the pressure applied to plunger, W the resistance overcome by the ram, then

$$\mathrm{P} : \mathrm{W} = \pi s^2 : \pi r^2 = s^2 : r^2.$$

If a force, Q, acting at an arm, a, of a lever produces a pressure, P, on the plunger at a distance, b, from the fulcrum, we have

$$\mathrm{P}b = \mathrm{Q}a, \therefore \mathrm{W} = \frac{\mathrm{Q}a}{b} \times \frac{r^2}{s^2}.$$

The loss of power by the friction of the collars is not serious. Mr. Hick estimates that for rams of four inches in diameter the friction is about $1\frac{1}{2}$ per cent. of the total resistance, and for larger rams it is less in proportion.

196. **The accumulator** is a vessel in which water is confined and stored up under pressure, and from which it may be drawn off to work hydraulic machinery. It is apparent that if water be pumped or forced into a vessel fitted with a movable plunger, and, if the plunger be loaded with a sufficient weight, the enclosed water will be in the same mechanical condition as if it were in communication

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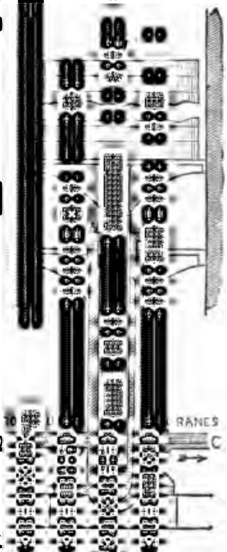

[illegible] A B is [illegible] up by [illegible] found [illegible] weight [illegible]

[illegible] area [illegible] lbs. [illegible] water [illegible]

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The phenomenon on which we make a few observations is commonly known as *capillary* attraction or repulsion. The term 'capillary' attraction denotes rather the effect observed than the cause of it, and is applied to express what happens in the wick of a lamp where oil is continually drawn up to supply the flame from a liquid reservoir below it.

The spirit lamp which is used to heat water in an urn at a breakfast-table is often a small bundle of fine wires bound round by a piece of wire gauze, the liquid being sucked up by the small tubes or passages existing between the solid wires.

In order to exhibit capillary action, take two tubes of glass with bores of (say) $\frac{1}{30}$ and $\frac{1}{60}$ of an inch in diameter. Wash them carefully with concentrated sulphuric acid, and rinse them with distilled water, which will now thoroughly wet the glass.

It will be found that the liquid rises on the outside of each tube a very little, and that it rises sensibly in the inside, and to a higher point in the smaller tube. It would appear as if the glass attracted the water, the attraction being more powerful the smaller the diameter of the tube. Such an explanation has been commonly accepted, but it is now considered insufficient, and the phenomenon is referred to an action which takes place on the surface of the liquid. The law according to which the liquid rises in the tube has been verified by many experiments, and is the following :—

198. **Capillary law, or law of diameters.**—When a liquid rises by capillary action in a tube, the height to which it ascends is inversely as the diameter of the tube. The meaning of the law is this, that if water rises to a height of 30 units in a tube of diameter 1, it will rise to a height of 15 units in a tube of diameter 2, and to a height of 300 units in a tube of diameter $\frac{1}{10}$. The less the diameter the greater the height, and in the same proportion as the diameter diminishes.

The length of a *metre* is 39·37 inches, the metre being the French measure of length closely corresponding to our yard measure.

A *millimetre* is $\frac{1}{1000}$ part of a metre, or is ·03937 inch, or about $\frac{1}{25}$ of an inch. In the above statement let the unit to which the numbers refer be 1 millimetre.

Then 30 millimetres = 1·18 inch.
15 " = ·59 "
1 " = ·039 "
2 " = ·078 "

That is to say, in a tube about $\frac{1}{25}$ of an inch in diameter, the water would rise $\frac{6}{5}$ of an inch, and in a tube $\frac{2}{25}$ of an inch in diameter it would rise $\frac{3}{5}$ of an inch, whereas in a tube $\frac{1}{250}$ of an inch in diameter it would rise to a height of nearly 1 foot.

These numbers are approximately true, as will appear by the following experiment made by Gay Lussac, one of the fathers of chemistry :—

Diameter of tube in millimetres	Height of column of water in millimetres
1·29	23·16
1·90	15·58

But if height of column is inversely proportional to the diameter of the tube, we ought to have
height of column × diameter of tube = a constant number.
And in this case—

$$1{\cdot}29 \times 23{\cdot}16 = 29{\cdot}87$$
$$1{\cdot}90 \times 15{\cdot}58 = 29{\cdot}60$$

which numbers are very nearly equal, as they ought to be.

The next point is that, under the above circumstances, water rises higher than any other liquid. But the capillary law, or the *law of diameters*, holds equally in all cases. Thus Gay Lussac found, when experimenting with alcohol—

Diameter of tube in millimetres	Height of column of alcohol in millimetres
1·29	9·18
1·90	6·08

And we observe that—

$$1{\cdot}29 \times 9{\cdot}18 = 11{\cdot}84$$
$$1{\cdot}90 \times 6{\cdot}08 = 11{\cdot}55$$

which products are very nearly equal, or the law of diameters is experimentally verified with alcohol.

If two plates of glass, which are held together tightly along an edge, and somewhat separated away from the edge, are dipped into a vessel of coloured water, the water will rise in a film with a curved edge. The plates should be most carefully cleaned and wetted in the first instance.

199. Capillary repulsion is exhibited in the case of mercury, which does not, like water, wet the surface of glass, and accordingly we find that, if the glass tubes used in the first experiment be dipped into mercury, the liquid is not raised or attracted, but depressed or repelled, and the depression is greater in the smaller tube. In this case there is a film of air, no doubt, between the mercury and the tube, whereby the law of diameters is interfered with.

EXAMPLES.

Science Exam. 1885.—1. Two rods of uniform density are put together, so that the one stands on the middle point of the other and at right angles to it; the former weighs 3 lbs. and the latter 2 lbs.; find the centre of gravity of the whole.

Ans. Let A B be the longer rod, then A G = $\frac{3}{10}$ A B.

2. Forces of 7 and 16 units have a resultant of 21 units; find the directions of the forces by a construction drawn to scale. *Ans.* Angle between the forces = 52° 37′.

3. A and B are two fixed points 5 inches apart in a horizontal line; a rod, A C, 2 feet long is fastened to A by a hinge, and rests on B; if a weight of 5 lbs. is hung at C, what pressures does it produce at A and B?

Ans. Pressure at A = 19 lbs., and pressure at B = 24 lbs.

4. If the modulus of elasticity of cast steel is 31,500,000 (in pounds per square inch of section), and if a steel rod 14 yards long and 7 square inches in section is held at one end, what force must be applied in the direction of its length at the other end to stretch it one-tenth of an inch? *Ans.* 43,750 lbs.

5. If a man does 1,056,000 foot-pounds in a working day of 8 hours, at what fraction of a horse power does he work on the average? *Ans.* $\frac{1}{15}$ horse power.

6. A train is moving at the rate of 54 miles an hour; at what rate is it moving in feet per second?

Ans. $79\frac{1}{5}$ feet per second.

7. A body is thrown upward with a velocity of 110 feet per second: after how many seconds will it be moving downward with a velocity of 66 feet per second, assuming $g = 32$?

Ans. $5\frac{1}{2}$ seconds.

8. A body weighing 200 lbs. is moved from a state of rest, and is found subsequently to be moving at the rate of 12 feet a second. How many units of work must have been done on it by the forces causing motion over and above those expended on the resistances? Give the answer in foot-pounds, assuming $g = 32$. *Ans.* 450 foot-pounds.

9. If the plane has a height of 3 feet for every 10 feet of its length, what distance would a particle descending along it, from a state of rest, describe in 2½ seconds?

Here $f = 9{\cdot}6$, $s = \frac{1}{2}ft^2 = 30$ feet.

10. A cube, each of whose edges is 2 feet long, stands on one face on the bottom of a vessel containing water 4 feet deep; find the pressure of the water on one of the upright faces of the cube, assuming that a cubic foot of water weighs 1000 oz.

Ans. Pressure $= 3 \times 4 \times 1000$ oz. $= 750$ lbs.

11. Given that the specific gravity of petroleum is 0·88, and that a quart of water weighs 40 oz., find how many gallons of petroleum will weigh 38½ lbs. *Ans.* 4⅜ gallons.

12. If the plunger of a force-pump has a cross-section of 4 square inches, and works 80 feet below a cistern, with what pressure must it be pushed down in order to force water into the cistern?

$$\text{Here total pressure} = \frac{4 \times 80 \times 125}{144 \times 2} \text{ lbs. per square inch.}$$
$$= 138\tfrac{8}{9} \text{ lbs.}$$

13. A uniform bar of metal 10 inches long weighs 4 lbs., and a weight of 6 lbs. is hung from one end. Find the fulcrum or point upon which the bar will balance.

Let x be distance of fulcrum from the end at which the weight is hung. Then $4\,(5 - x) = 6\,x$, $\therefore x = 2$ inches.

14. A horse drawing a cart along a level road at the rate of 2 miles per hour performs 29,216 foot-pounds of work in 3 minutes. What pull in pounds does the horse exert in drawing the cart? *Ans.* 55⅓ lbs.

15. What must be the diameter in inches of a round rod of wrought-iron in order to sustain a load of 50 tons? It is given that a bar of iron 1 square inch in section will just support a load of 25 tons. *Ans.* 1·595 inch.

16. A rectangular tank has an inclined base ABCD, whereof the side AB is 4 feet long, and each point of it is 4 feet below the surface of the water, while the side CD is also 4 feet long, each point of it being 2 feet below the surface of the water. Find the pressure in pounds upon the base, it being given that AD = BC = 6 feet, and also that a cubic foot of water weighs 62½ lbs.

Ans. Pressure on base $= 6 \times 4 \times 3 \times 6{\cdot}25 = 4{,}500$ lbs.

17. Given that a rectangular rod of fir, 10 inches long, 1 inch broad, and 1 inch deep, and supported at both ends, will just sustain 540 lbs. when hung at its centre, what should be the depth of a bar of like wood 5 feet long and 2 inches broad, and supported at both ends, in order to support a load of ¾ of a ton when hung at its centre? *Ans.* 3·055 inches.

18. A weight of 400 lbs. is being raised by a pair of pulley blocks, each having two sheaves. The standing part of the rope is fixed to the upper block, and the parts of the rope, whose weight may be disregarded, are considered to be vertical. Each block weighs 10 lbs. What is the pressure at the point from which the upper block hangs? Sketch the arrangement. *Ans.* 522½ lbs.

19. Sketch a sectional elevation of a hydraulic-press, showing the mode of packing the ram. The power which actuates the force-pump is applied at the end of a lever giving a mechanical advantage of 14 to 1, and the area of the plunger of the pump is 1 square inch. What power must be applied to the end of the lever to produce a pressure of 1 ton per square inch on the water enclosed in the press? *Ans.* 160 lbs.

20. A compound axle consists of two parts, the diameters being 10 and 12 inches respectively, and a rope is coiled round them in opposite directions so as to form a loop, upon which hangs a pulley loaded to 48 lbs. Considering the parts of the rope to be vertical, find the force which, acting at a leverage of 4 feet upon the axle, will just balance the weight. Sketch the arrangement. *Ans.* ½ lb.

21. A wheel of 40 teeth is driven by a winch-handle 14 inches long, and gears with a rack having teeth of 1-inch pitch; apply the principle of work to find the driving pressure exerted

on the rack when a power of 50 lbs. is applied at the end of the winch-handle. *Ans.* 110 lbs.

22. The rods of a double-barrelled pump are attached to a double-cranked shaft, at the end of which is a wheel with 30 teeth. This wheel gears with a pinion of 8 teeth driven by a winch-handle. Find the number of single strokes performed by each pump-rod while the winch-handle makes 15 revolutions, and sketch the arrangement. *Ans.* 8 single strokes.

Science Exam. 1884.—1. In a certain state of the atmosphere 100 cubic inches of air weigh 31 grains; at a temperature the same as that of the air 30 cubic inches of mercury weigh 14·88 lbs.; find the number of cubic inches of air which contain as much matter as a cubic inch of mercury.

Ans. 11,200 cubic inches.

2. A circular lamina weighs 9 oz.; a thin straight wire as long as the radius of the circle weighs 7 oz. If the wire be placed on the lamina so as to be a chord of the circle, find the centre of gravity of the whole.

Let AB be the wire, C its centre, also let O be the centre of the circle, and G the required centre of gravity.

Then $9 \times OG = 7 \times CG$. Also $OG + CG = \sqrt{\frac{3}{2}} \times$ radius of circle.

Whence $OG = \cdot3788 \times$ (radius of circle).

3. Find (by construction or otherwise) the resultant of three forces, viz. 12 and 15 units acting in opposite directions along parallel lines, and 8 units acting along a line at right angles to the former lines. Resultant $= \sqrt{64 + 9} = 8\cdot544$ units.

4. A force of 50 units acts along a line inclined at an angle of 30° to the horizon; find, by construction (or otherwise), its horizontal and vertical components. The force and the two components just found are, of course, in a vertical plane; find next the components of the force of 50 units along two lines in that plane at right angles to each other, and inclined at angles of 45° to a horizontal line.

Ans. The components are

(1) 25, 43·3 units.
(2) 48·3, 12·9 units.

5. A square board, ABCD, is capable of turning freely in its plane, which is vertical, round A; it is held so that AB is vertical by a force acting along BC; find the reaction of the fixed point at A and the force along BC in terms of the weight of the board; find also the line along which the reaction acts.

Ans. Reaction at A $= \frac{W\sqrt{5}}{2}$.

Force along BC $= \frac{W}{2}$.

The reaction passes through the middle point of BC.

6. A cylinder, whose height is three times the diameter of its base, stands on a plane horizontal at first, but afterwards inclined to the horizon at a gradually increasing inclination; supposing the friction sufficient to prevent sliding, find, by construction or otherwise, at what inclination the cylinder will topple over. If a cone, whose height and base are equal to those of the cylinder, had been placed on the same plane, why would the cone stand after the cylinder had toppled over?

Ans. Angle at which cylinder upsets is 18° 26′.
" " cone " 33° 41′.

7. A body weighing 10 lbs. is placed on a horizontal plane, and is made to slide over a distance of 50 feet by a force of 4 lbs.; what number of units of work is done by the force? If the coefficient of friction between the body and the plane is 0·3, what number of units of work is done against friction?

Ans. 200 foot-pounds.
150 "

8. A force F, acting on a mass of 5 lbs., increases its velocity in every second by 12 feet a second; a second force P, acting on a mass of 28 lbs., increases its velocity in every second by 7½ feet a second; find the ratio of F to P.

Ans. The ratio is 2 to 7.

9. A small heavy ball, suspended by a fine thread 6 ft. 4½ in. long, was found to make 151 oscillations in 3 m. 31 s.; calculate the accelerative effect of gravity at the place where the experiment was made. *Ans.* 32·223.

10. A rectangular hole, ABCD, whose lower side, CD, is horizontal, is made in a side of a reservoir and is closed by a door whose plane is vertical; the door can turn freely outward round

a hinge coinciding with CD; calculate the force that must be applied to AB to keep the door shut, assuming that AB is 1 foot and AD 12 feet long, and that the water rises to the level of AB.

Let P = pressure on door, Q = required force.
Then $4\text{P} = 12\text{Q}$, also $\text{P} = 36 \times 125$, $\therefore \text{Q} = 1500$ lbs.

11. The standard weight of a Nicholson's hydrometer is 1,250 grains; a small substance is placed in the upper pan, and it is found that 530 grains are needed to sink the instrument to the standard point; but when the substance is put into the lower pan, 620 grains are required; what is the specific gravity of the substance? *Ans.* 8.

12. One foot of the length of the barrel of the pump holds a gallon of water (10 lbs.); at each stroke the piston works through 4 inches; the spout is 24 feet above the surface of the water in the well; how many foot-pounds of work are done per stroke? *Ans.* 80 foot-pounds.

13. A car laden with 20 passengers is drawn up an incline, one end of which is 160 feet above the other; the car, when empty, weighs 2 tons, and the average weight of each passenger is 140 lbs. Find the number of foot-pounds of work done in ascending the incline, neglecting friction. *Ans.* 1,164,800 foot-pounds.

14. A uniform beam, 12 feet long and weighing 100 lbs., is supported at both ends and carries a weight of 2 cwt. at a distance of 3 feet from one end; find the pressure on each point of support. *Ans.* 218 lbs. and 106 lbs.

15. A weight of 4 cwt. is raised by a cord which passes round a drum 3 feet in diameter, having on its shaft a toothed wheel also 3 feet in diameter. A pinion, 8 inches in diameter, and driven by a winch-handle 16 inches long, gears with the wheel. Find the power to be applied to the winch-handle in order to raise the weight. *Ans.* 112 lbs.

16. The diameter of the pitch circle of a pinion working into a rack is 10 inches. How far will the rack travel for two revolutions of the pinion? *Ans.* 20π, or 62·83 inches.

17. Find the dimensions of a transverse section of a square rod of fir to sustain a suspended load of 10 tons, the rod being

held vertically. The breaking load of a rod of fir one square inch in section is 6 tons.

Ans. Side of square = 1·29 inch.

18. A beam, 20 feet long, whose weight is neglected, is supported at both ends and loaded with 1 ton evenly distributed along its length. Find the bending moment at a distance of 7 feet from one end.

Ans. 5,096, the units being in feet and pounds.

19. A pulley, 24 inches in diameter, makes 50 revolutions per minute, and transmits 3-H.P. by means of a strap running upon its rim. Find the tension of the strap.

Ans. 315·119 lbs.

20. The diameter of the plunger of a force-pump is 2½ inches and it is driven by a crank 2 inches in length making 30 revolutions per minute; find the number of cubic inches of water pumped up in 5 minutes. *Ans.* 2,945¼ cubic inches.

21. A vertical bar, moving in guides, is driven by a circular cam-plate having a centre of motion in the line of direction of the bar. The distance from the centre of motion to the centre of the plate is 2 inches, and the bar exerts a pressure of 10 lbs. when rising, but falls by its own weight. Find the work done in 100 revolutions of the plate. *Ans.* 333⅓ foot-pounds.

22. A cylindrical can is 6 inches in diameter and 36 inches deep, and is required, when empty, to stand in a bath of water 30 inches deep without being lifted up. To what weight must the can be loaded, the weight of a cubic foot of water being 62½ lbs. *Ans.* 30·68 lbs.

Science Exam. 1883.—1. If 100 cubic inches of oxygen (under certain circumstances of pressure and temperature) weigh 35 grains and 1 cubic inch of mercury weighs ·49 lb., how many cubic inches of oxygen would contain the same quantity of matter as 1 cubic inch of mercury? *Ans.* 9,800 cubic inches.

2. Two bodies move with constant velocities, one describes 36 miles in 1 h. 20 m. and the other 55 feet in 1½ sec.; compare the two velocities. If the former body weighs 50 lbs. and the latter 72 lbs., compare their momenta.

Ans. Velocities are as 9 to 10.
Momenta " 5 " 8.

3. If a square tin plate weighs 5 oz. and a small body weighing 2 oz. is placed at one corner of the plate, find the centre of gravity of the whole.

Ans. At a point in the diagonal through the weighted corner distant $\frac{5}{14}$ of the diagonal from the corner.

4. Three forces act at a point and keep it at rest, show how to draw a triangle whose sides shall represent the forces. AC and BC are two threads, 4 feet and 5 feet long respectively, fastened to fixed points A and B, which are in the same horizontal line, 6 feet apart: a weight of 50 lbs. is fastened to C; find, by means of a construction drawn to scale, the pull it causes at the points A and B. *Ans.* Tension of AC = $37\frac{3}{4}$ lbs.
„ BC = $28\frac{1}{3}$ „

5. A rod AB, whose length is 5 feet and weight 10 lbs., is found to balance itself if supported on a fulcrum 3 feet from A. If this rod were placed horizontally on two points, one under A and the other under B, what pressure would it exert on each point? *Ans.* 4 lbs. and 6 lbs.

6. A lever (AB) of the first order, 8 feet long, has the fulcrum 2 feet from B; a weight of 5 lbs. is hung from A, and one of 17 lbs. from B; putting the weight of the lever itself out of the question, from what point must a weight of 2·5 lbs. be hung to keep the lever horizontal? *Ans.* $3\frac{3}{8}$ feet from B.

7. Weights of 10 lbs. and 8 lbs. are connected by a very fine thread which rests on a rough fixed pulley, so that they hang suspended; the heavier weight is found to be just not heavy enough to fall and draw the lighter weight up; if now we suppose the weights made to move uniformly, so that one goes up and the other down through 12 feet, how many foot-pounds of work are done against friction during the motion? Would this amount of work be done against friction if the friction were such that in the descent the bodies moved faster and faster?

Ans. 24 foot-pounds.

8. A body, known to be acted on by a constant force, begins to move from a state of rest; it is observed to move through 55 feet in a certain two seconds, and through 77 feet in the next

two seconds; what distance did it describe in the first six seconds of its motion? Here $f = 5$ feet per second.
Distance required $= 90$ feet.

9. A mass of $6\frac{3}{4}$ lbs. is put on a smooth horizontal table and connected by a fine thread to a mass of $1\frac{1}{4}$ lb., which hangs over the edge of the table; if the latter body is allowed to fall, dragging the former after it, what force does it exert on the former body, and what is its accelerative effect on the velocity of that body? *Ans.* $1\frac{7}{128}$ lb.; 5 feet per second.

10. The two parallel legs of a bent tube are held vertically, and liquids which do not mix are poured slowly into the tube, the heavier being poured first; find in what position they come to rest.

If the two liquids are water and oil (sp. gr. 0·88), and the water rises 11 inches above the common surface, to what height does the oil rise?

Ans. The heights above the common surface are inversely as the densities. Height of oil = 12·5 inches.

11. A cylinder is fitted with a piston which works in it easily and air-tight; when the water barometer is at 33 feet, the piston is 1 foot above the bottom of the cylinder; if the cylinder were slowly sunk in water to a depth of 44 feet, how high would the piston be above the bottom of the cylinder?

Ans. $5\frac{1}{7}$ inches.

12. If the pressure in the receiver of an air-pump were reduced to $\frac{1}{3}$ of the atmospheric pressure in four strokes, to what would it be reduced in six strokes?

Ans. ·1924 of the atmospheric pressure.

13. How is the work done by a force measured? The resistance to traction on a level road is 150 lbs. per ton of weight moved; how many foot-pounds of work are expended in drawing 6 tons through a distance of 150 yards?

Ans. 405,000 foot-pounds.

14. Explain the manner in which the principle of work is applied in determining the relation of P to W in the lever. A lever, centred at one end, is 15 feet long, and a weight of W lbs.

hangs from the opposite end. The weight W is supported by an upward pressure of 28,270 lbs. at 13 feet from the fulcrum. Find W. *Ans.* $24,500\frac{2}{3}$ lbs.

15. In a lifting crab the lever-handle is 14 inches long, the diameter of the drum is 6 inches, and the wheel and pinion have 57 and 11 teeth respectively. Find the weight in pounds which could be raised by a force of 50 lbs. applied to the lever-handle, friction being neglected. *Ans.* $1,209\frac{1}{11}$ lbs.

16. A sluice-gate is 4 feet broad and 6 feet deep, and the water rises to a height of 5 feet on one side and 2 feet on the other side. Find the resultant pressure in pounds on the gate. *Ans.* 2,625 lbs.

17. If the diameter of the ram of a hydraulic-press is 9 times that of the force-pump, and if P be the pressure on the pump, what is the pressure exerted by the ram, neglecting friction? *Ans.* 81 P.

18. In a compound wheel and axle the diameters of the two parts of the axle are 5 and 6 inches respectively. The weight raised, viz. W, hangs from a single movable pulley in the usual manner, and is supported by a pressure P applied perpendicularly to a lever-handle 15 inches in length. Find the ratio of P to W. *Ans.* $W = 60P$.

19. Friction is neglected, and it is found that a force acting horizontally will move 10 lbs. up 5 feet of an incline rising 1 in 4. Find the work done, and find also the force parallel to the plane which will just support the weight of 10 lbs. *Ans.* $12\frac{1}{2}$ foot-pounds, $2\frac{1}{2}$ lbs.

20. A rectangular beam of timber is supported at both ends, and can just bear a weight of W lbs. evenly distributed without breaking. If the load were all brought into the centre, how much should the breadth of the beam be increased, the depth remaining unchanged? *Ans.* The breadth should be doubled.

21. A rectangular beam of fir, of uniform section throughout, is supported horizontally on two walls 15 feet apart, and has to carry a load of $1\frac{1}{2}$ ton at 5 feet from one of the walls. The width of the beam is 5 inches; find its depth, taking the safe load at one-fourth the breaking load. How much should the

depth of the beam be increased, the breadth remaining constant, if the load were shifted from its original position to the centre of the beam—the breaking weight of a beam of fir 15 inches long, 1 inch broad, and 1 inch deep, supported at both ends and loaded in the middle, being taken at 360 lbs.?

Ans. Depth = 8·9 inches.
Increased depth = 9·465 inches − 8·9 inches.

22. The resistance of friction along an inclined plane is taken at 150 lbs. for each ton of weight moved. Find the work done in drawing 2 tons up 100 feet of an incline which rises 1 foot in height for 25 in length. *Ans.* 47,920 foot-pounds.

23. Explain the mechanical advantage resulting from the employment of an endless screw and worm wheel. The lever-handle which turns an endless screw is 14 inches long, the worm wheel has 32 teeth, and a weight W hangs by a cord from a drum of 6 inches in diameter, whose axis coincides with that of the worm wheel. If a pressure P be applied to the lever-handle, find the ratio of P to W.

Ans. P : W : : 3 : 448.

Science Exam. 1882.—1. Define Mass and Density. Of two bodies one has a volume of 5 cubic inches, the other of one-fifth of a cubic foot. In a perfectly just balance the former weighs 15 oz., the latter 12·8 lbs.; what is the ratio of the mass of the first to that of the second, and what is the ratio of the density of the first to that of the second?

Ans. Ratio of masses is 75 to 1024.
„ densities 81 „ 16.

2. Define the resultant of two forces. Forces of 5 and 12 units respectively act at a point; what are their directions when their resultant is greatest, and what when it is least? Find the resultant when greatest, when least, and when the forces act at right angles to each other. *Ans.* 17, 7, 13 units.

3. Define the moment of a force with respect to a point. If two forces balance each other on a weightless rod capable of turning freely round a fixed point, what relation must exist between the forces? C is a point in a weightless rod (AB) round which it is capable of turning freely; AC is one-third of AB; a force of 10 units acts at A perpendicularly to AB, and is

balanced by an equal force acting at B; find how the second force must act, and the magnitude and direction of the pressure on the fulcrum.

Ans. 30° to rod.
$10\sqrt{3}$ units at an angle of 60° to rod.

4. A particle is found to be moving in a straight line at the rate of 5 feet a second; a quarter of a minute afterwards, at the rate of 50 feet a second; half a minute afterwards, at the rate of 95 feet a second. Show that this is consistent with the action of a constant force on the body, and find the ratio which that force bears to the force of gravity on the body.

Ans. The ratio is 3 to 32.

5. A body whose weight is 12 lbs. and specific gravity 2·5 is placed in a vessel with horizontal base containing water; what pressure is exerted on the base by the body, supposing the body to be quite covered with water? *Ans.* $7\frac{1}{5}$ lbs.

6. A hollow cylinder is full of air at a pressure of 15 lbs. per square inch when the piston is 12 inches from the bottom; if more air is forced in till there is three times as much air as at first, and if the piston is allowed to rise 4 inches, what is now the pressure of the air per square inch (temperature constant)?

Ans. $33\frac{3}{4}$ lbs. per square inch.

7. A ball, weighing 100 lbs., rests on an inclined plane, being held in position by a string which is fastened to a bracket so as to be parallel to the plane. The height of the plane being one-third of the length, find the tension of the string and the pressure perpendicular to the plane. Establish your results by reasoning on known principles, such as the principle of work or that of the parallelogram of forces. *Ans.* $33\frac{1}{3}$ lbs., 94·28 lbs.

8. A hole is punched through a plate of wrought-iron one half inch in thickness, and the pressure actuating the punch is estimated at 36 tons. Assuming that the resistance to the punch is uniform, find the number of foot-pounds of work done.

Ans. 3,360 foot-pounds.

9. The area of the plunger of a force-pump being 3 square inches, find the pressure upon it when water is forced up to a height of 20 feet. *Ans.* $26\frac{1}{24}$ lbs.

10. In the headstock of a lathe a pinion of 20 drives a wheel of 60, and a second pinion of 20 drives another wheel of 60; compare the rates of rotation of the first driving pinion and of the mandril of the lathe. Sketch the arrangement.

Ans. 9 to 1.

11. A punching-machine is provided with a fly-wheel and driven by an engine at such a rate that two holes are punched in three minutes. The plate operated on is 1 inch thick, and it is estimated that a mean pressure of 69 tons is exerted through the space of one inch. Find the average amount of work done per minute by this machine.

Ans. ·26 horse-power, or 8,586⅔ foot-pounds.

12. In the transmission of power by a rope, the wheel carrying the rope is 14 feet in diameter and makes 30 revolutions per minute, the tension of the rope being 100 lbs. Find the amount of power transmitted as estimated in horse-power.

Ans. 4 horse-power.

13. What must be the breadth in inches of an oak cantilever or overhanging beam, 6 feet long and 9 inches deep, in order to carry a load of ½ ton at its extremity, and how much must its breadth be increased in order that it may carry an additional load of ½ ton uniformly distributed over its length. The actual stress is not to exceed ¼ of the breaking stress, and the breaking weight of an oak cantilever 6 inches long, 1 inch deep, and 1 inch broad is 280 lbs. *Ans.* 2·37 inches.

Additional breadth = 1·18 inch.

Science Exam. 1881.—1. Forces of 5 and 7 units act in the same direction along parallel lines at points 2 feet apart. Where is their centre? If the direction of the former force is reversed, where will now be their centre? Show the position of the centre in each case by a diagram.

Ans. 14 inches from the force 5.
84 „ „ „ 5.

2. A pumping-engine is partly worked by a weight of 2 tons, which at each stroke of the pump falls through 4 feet; the pump makes 10 strokes a minute. How many gallons of water (1 gallon weighing 10 lbs.) are lifted per minute by the weight from a depth of 200 feet?

N.B.—You need not concern yourself with the question how the weight is lifted between the strokes.

Ans. 89·6 gallons.

3. What is meant when it is said that the acceleration of the velocity of a particle is 10, the units being feet and seconds? If the particle were moving at any instant at the rate of 7½ feet a second, after what time would its velocity be quadrupled? And what distance would it describe in that time?

Ans. 2¼ seconds, $42\frac{3}{16}$ feet.

4. A rod suspended by a fine axis passing through one end of it makes 66 oscillations a minute; find another point in it from which, if it be suspended, it will still make 66 oscillations a minute ($g = 32$). *Ans.* 2·69 feet from the end.

5. Two equal small areas (A and B) are marked on a side of a reservoir at different depths below the surface of the water; what is the ratio of the pressure of the water on A to its pressure on B?

The pressure on A is four times the pressure on B; but if water is drawn off, so that the surface of the water in the reservoir falls a foot, the pressure on A is now five times the pressure on B. At what depth were A and B below the surface in the first instance? *Ans.* Depth of A = 16 feet.
,, B = 4 feet.

6. A rod of uniform cross section, 18 inches long, weighs 3 oz.; its specific gravity is 8·8; what fraction of a square inch is the area of its cross section? The weight of a cubic inch of water may be taken to equal 252 grains.

Ans. ·03288 square inch.

7. A barometer stands at 30 inches, the vacuum above the mercury being perfect; the area of the cross section of the tube is ¼ of a square inch. If ¼ of a cubic inch of the external air is allowed to get into the barometer and the mercury is found to fall 4 inches, what was the volume of the original vacuum?

Ans. $\frac{7}{8}$ cubic inch.

8. A pump is worked directly from the ram of a water-pressure engine, the cylinder of which is 6 inches in diameter, that of the pump being 8¾ inches. The head of water in the supply

pipe which gives the power is 450 feet, and that in the delivery pipe is 150 feet; find the ratio of work done to power expended. *Ans.* 1 to 1·41.

9. The plunger of a force-pump is $8\frac{3}{4}$ inches in diameter, the length of the stroke is 2 feet 6 inches, and the pressure of the water is 50 lbs. per square inch; find the number of units of work done in one stroke. *Ans.* 7516·5 foot-pounds.

10. A lock-gate is 12 feet wide, and the water rises to a height of 8 feet from the bottom of the gate. What pressure in pounds does it sustain? (The weight of a cubic foot of water is $62\frac{1}{2}$ lbs.) *Ans.* 24,000 lbs.

11. A valve, 3 inches in diameter, is held down by a lever and weight, the length of the lever being 10 inches, and the valve spindle being 3 inches from the fulcrum. You are to disregard the weight of the lever and to find the pressure per square inch which will lift the valve when the weight hung at the end of the lever is 25 lbs.

Ans. 11·79 lbs. above the atmosphere.

12. Define the pitch circle of a toothed wheel. Two parallel axes are at a distance of 10 inches, and they are to rotate with velocities as the numbers 2 and 3 respectively. What should be the diameters of the pitch circles of a pair of wheels which would give the required motion? *Ans.* 12 inches and 8 inches.

13. A weight of 20 lbs. draws up W lbs. by means of a wheel and compound axle. The diameter of the wheel is 5 feet, and the diameters of the parts of the compound axle are 9 and 11 inches respectively; find W. *Ans.* 1,200 lbs.

14. State the rule for finding the amount of work stored up in a given weight when moving with a given velocity. A weight of 6 cwt. moves with a velocity of 20 feet per second; how many units of work are stored up in it?

Ans. Taking $g = 32$, the answer is 4,200 foot-pounds.

15. The rim of a fly-wheel weighs 9 tons, and the mean linear velocity of its mass is assumed to be 40 feet per second; how many foot-tons of work are stored up in it? If it be required to store the additional work of 9 foot-tons, what should be the increase of velocity?

Ans. 223·6 foot-tons, 8 feet per second.

16. A bar of wood 7 feet long and 2 inches square is supported at both ends and is broken by a weight of 500 lbs. suspended at the centre. What weight in pounds will a rectangular bar of the same material, supported and loaded in like manner, sustain when its length is 8 feet, its breadth 2½ inches, and its depth 4 inches? *Ans.* 2187·5 lbs.

Science Exam. 1880.—1. How is the quantity of matter in a given body ascertained? One body (A) has a volume of 1·35 cubic foot and a specific gravity of 4·4; a second body (B) has a volume of 10·8 cubic inches and a specific gravity of 19·8; what ratio does the quantity of matter in A bear to that in B?

Ans. 48 to 1.

2. Define the modulus of elasticity. A wire 10 feet long, whose cross section is $\frac{1}{100}$ of a square inch, is lengthened by $\frac{1}{20}$ of an inch when stretched by a weight of 68 lbs.; what is the modulus of elasticity of the material?

Ans. 16,320,000 lbs. per square inch.

3. Two parallel forces of 10 and 12 units act in opposite directions at points 2 feet apart; where is their centre? What is the distinctive property of the centre of two parallel forces?

Ans. 12 feet from force 10.

4. Define a foot-pound and a horse-power. What is the horse-power of a waterfall of 18 feet, when the stream above the fall passes through a section of 6 square feet at the rate of 2½ miles an hour? *Ans.* 45 H.P.

5. What is the acceleration of the velocity of a moving body? How is it measured when constant? A body, whose velocity is known to be uniformly accelerated, is moving at the rate of 100 yards a minute, and 10 seconds afterwards at the rate of 160 yards a minute; what is the acceleration of its velocity, and what distance does it describe in those 10 seconds?

Ans. $\frac{3}{10}$ of a foot per second, 65 feet.

6. Two particles whose masses are 5 lbs. and 3 lbs. are connected by a perfectly flexible weightless thread which rests on a smooth point. The heavier mass draws up the lighter; when it has fallen through 5 feet, what is the kinetic energy of the particles and their velocity? ($g = 32$).

Ans. 10 foot-pounds, 8·944 feet per second.

7. Describe briefly the mercurial gauge of an air-pump. If the barometer stands at 29·7 inches, what will the gauge read when the quantity of air withdrawn is ten times as much as the quantity left in the receiver? *Ans.* 2·7 inches.

8. An endless cord, stretched, and running over grooved pulleys with a linear velocity of 3000 feet per minute, transmits five horse-power; find the tension of the cord in pounds.
Ans. 55 lbs.

9. What is the unit of work? A horse drawing a cart at the rate of 2 miles per hour exerts a traction of 156 lbs.; find the number of units of work done in 1 minute.
Ans. 27,456 foot-pounds.

10. In the force-pump of a press the area of the plunger is $\frac{1}{3}$ of a square inch, the distance from the fulcrum of the lever-handle to the plunger is 2 inches, and the distance from the fulcrum to the other end of the lever is 2 feet; what pressure per square inch is exerted on the water underneath the plunger when a weight of 20 lbs. is hung at the end of the lever-handle?
Ans. 720 lbs.

11. In a crane there is a train of wheelwork, the first pinion being driven by a lever-handle and the last wheel being on the same axis as the chain-barrel of the crane. The wheelwork consists of a pinion of 11 gearing with a wheel of 92, and of a pinion of 12 gearing with a wheel of 72, the diameter of the barrel being 18 inches and that of the circle described by the end of the lever-handle being 36 inches; find the ratio of the power to the weight raised, friction being neglected.
Ans. P : W :: 11 : 1104.

12. A rectangular beam of timber supported at both ends, and of a given breadth and depth, just supports a load W at its centre. If the load be shifted to a point halfway between the centre and one end, how much may the depth be reduced?
Ans. 13·4 per cent.

13. A horizontal screw, of one inch pitch, is fitted to a sliding nut which is pulled horizontally by a cord passing over a fixed pulley, and having a weight W attached to it. To the free end of the screw there is fixed a pulley of 20 inches diameter, from

the circumference of which a weight W hangs by a cord. Sketch the combination, and find P to W.

Ans. P : W :: 1 : 62·8.

14. What must be the effective horse-power of a locomotive engine which moves at the steady speed of 40 miles per hour on a level rail, the resistance being 15 lbs. per ton, and the weight of the engine and train being 100 tons? If the rails were laid at a gradient of 1 in 100 what additional horse-power would be required? *Ans.* 160 H.P.

Additional H.P. = 238·93.

15. In a direct-acting engine the length of the crank is 2 feet and that of the connecting-rod is 9 feet. How far is the piston from the middle of its stroke when the crank has moved through 60° from either dead point in the forward and backward strokes respectively? *Ans.* 9·98 inches, 14·02 inches.

Science Exam. 1879.—1. Define momentum. Of two bodies one has a mass of 2 cwt., and moves at the rate of 4 feet a second; the other has a mass of 21 lbs., and moves at the rate of 120 yards a minute: find the ratio of the momentum of the former body to that of the latter. *Ans.* 64 to 9.

2. A rod of uniform section and density, weighing 3 lbs., rests on two points, one under each end; a movable weight of 4 lbs. is placed on it. Where must this weight be placed that one of the points may sustain a pressure of 3 lbs. and the other point a pressure of 4 lbs.?

Ans. At a distance $\frac{5}{8}$ the length of rod from one end.

3. Find the velocity acquired by a body that falls freely from rest for a quarter of a second, and the distance it describes in that time, assuming that the accelerative effect of gravity is 32 in feet and seconds. *Ans.* Velocity = 8 feet per second.

Space = 1 foot.

4. A small body, whose mass is 8 lbs., tied to one end of a fine thread, 6 feet long, swings backward and forward in the arc of a semicircle. What is its kinetic energy, and what its velocity as it passes through the lowest point of the semicircle?

Ans. 48 foot-pounds.

Velocity = 19·6 feet per second

5. A small heavy ball is allowed to roll down an inclined plane; it is found that at the ends of the first, second, and third of three equal intervals of time the distances it has described from the starting-points are proportional to the numbers 1, 4, 9. What inference can be drawn from this fact as to the force that has acted on the body?

If the ball described 2½ feet in the first second of its motion, what distance would it describe in the third second?

Ans. 12½ feet.

6. Describe briefly Bramah's press.

If the diameter of the smaller plunger is ¾ inch, and that of the larger plunger (or ram) 15 inches, and the arms of the lever-handle 2 inches and 2 feet respectively, what force must be applied to the end of the long arm of the lever to make the ram raise a weight of 10 tons? *Ans.* 4⅔ lbs.

7. Enunciate the principle of Archimedes. Does it hold good in the case of gases as well as liquids?

Assuming that atmospheric air at ordinary pressures and temperatures weighs 0·31 grain, and hydrogen 0·02 grain per cubic inch, what will be the weight of a bladder, in which are 300 cubic inches of hydrogen, that will just stay where it is placed, *e.g.* on a table? *Ans.* 87 grains.

8. If 8 cubic inches of a liquid whose specific gravity is 1·25 are mixed with 12 cubic inches of another liquid whose specific gravity is 1·125, what is the specific gravity of the mixture if neither shrinking nor expansion occurs? *Ans.* 1·175.

9. A horse, walking at the rate of 3 miles an hour, draws a load of 1 cwt. up a shaft by means of a rope passing over a pulley at the mouth of a shaft; estimate in foot-pounds the work done in 1 minute. *Ans.* 29,568 foot-pounds.

10. Apply the principle of work in solving the following question:—The lever-handle of a crab is 3 times the diameter of the drum, and the wheelwork consists of a pinion of 16 teeth driving a wheel of 80 teeth; what weight will be lifted by a power of 30 lbs. acting at the end of the lever-handle?

Ans. 900 lbs.

11. A smooth incline is 8 feet long, and the total vertical rise from the bottom to the top thereof is 2 feet. What amount

of work is performed in drawing a weight of 100 lbs. up 4 feet of the incline, and what is the least force which will do this work?
Ans. 100 foot-pounds, 25 lbs.

12. The crank of an engine is 2 feet in length, and the diameter of the fly-wheel is 10 feet, also the fly-wheel has teeth on its rim and drives a pinion 3 feet in diameter. If the mean pressure on the crank-pin be 7½ tons, what is the mean driving pressure on the teeth of the pinion? *Ans.* 3 tons.

13. Draw a sectional elevation of the working parts of a common force-pump. The leverage to the end of the handle is five times that to the plunger, and the area of the plunger is 5 square inches; what power at the end of the lever-handle will produce a pressure of 45 lbs. per square inch on the water within the barrel? *Ans.* 45 lbs.

14. Describe the construction of a vessel suitable for storing up a supply of water under pressure and intended for actuating hydraulic machinery. If the plunger of this vessel be 17 inches in diameter, what load will bring the pressure of the water to 700 lbs. per square inch? *Ans.* 158,886 lbs., or 71 tons, 46 lbs.

15. A rectangular batten of fir, 6 feet in length, 2 inches broad, and 3 inches deep, is supported at its ends and can sustain a weight of 1100 lbs. when hung at the centre. If the load were equally distributed instead of being hung at the centre, how much would the bar support? *Ans.* 2200 lbs.

16. In a model illustrating the action of an endless screw and worm-wheel, the handle which turns the screw is 10 inches long, the screw is double-threaded, and the worm-wheel has 24 teeth. On the axis of the worm-wheel is a drum 4 inches in diameter, round which a cord is coiled. What weight hanging on the cord would be supported by 14 lbs. at the end of the lever-handle if there were no loss by friction? *Ans.* 840 lbs.

17. In a blowing engine of the overhead-beam construction the area of the steam piston is 2,712 square inches and the mean pressure of the steam is 30 lbs., while the area of the piston of the blowing cylinder is 16,272 square inches. The leverage of the working beam is as 15 on the steam side to 20 on the opposite side; what is the pressure of the air as it leaves the blowing cylinder?
Ans. 3¾ lbs. above that of the atmosphere.

18. In a compound wheel and axle, where the weight hangs on a single movable pulley, the diameters of the two portions of the axle are 3 and 2 inches respectively, and the lever-handle which rotates the axle is 12 inches in length. If a power of 10 lbs. be applied to the end of the lever-handle, what weight can be raised? *Ans.* 480 lbs.

19. In a fly-press there are two weights, each of 60 lbs., placed at the ends of an arm which drives the screw; and the velocity of each weight at the instant of striking the blow is 10 feet per second. The die at the end of the screw moves through $\frac{1}{10}$ inch in coming to rest; what mean statical pressure does it exert on the metal subjected to the operation of stamping?

Taking $g = 32$, *Ans.* 22,500 lbs.

20. An engine is employed to drive a fan by means of pulleys and bands, the first driving-band passing over the fly-wheel of the engine. The train consists of (1) the fly-wheel A, (2) two pulleys B and C on one axis, (3) two other pulleys D and E on another axis, and (4) a pulley F, to the axis of which the fan is attached. The diameters of A, B, C, D, E, F are as 12, 3, 8, 2, 5, 1 respectively, and A makes 20 revolutions per minute. How many revolutions does F make per minute? *Ans.* 1600.

Science Exam. 1878.—1. If a cubic centimetre of water contained exactly a gramme of matter, what would be the quantity of matter—estimated in grammes—in a cubic foot of lead; a linear foot being 30·45 centimetres, and the specific gravity of lead being 11·445. *Ans.* 323,130 grammes.

2. State the rule for finding the centre of gravity of two heavy points; and apply it to find the centre of gravity of two points weighing 4 and 5 ozs. respectively, placed 3 feet apart.

Ans. 16 inches from the heavier body.

3. Two forces of 10 units each have a resultant of 5 units; find by a construction the angle between the directions of the two forces. *Ans.* 151°.

4. A thread 14 feet long is fastened to two points A and B which are in the same horizontal line and 10 feet apart; a weight of 25 lbs. is tied to the thread at a point P so chosen that AP is 6 feet and therefore BP is 8 feet long; the weight being thus suspended, find, by means of a construction or

otherwise, what are the tensions of the parts (A P and B P) of the thread. *Ans.* 20 lbs., and 15 lbs.

5. Two trains A and B, moving towards each other on parallel rails uniformly at the rate of 30 miles and 45 miles an hour respectively, are 5 miles apart at a given instant; how far apart will they be at the end of 6 minutes from that instant; and at what distances are they from the first position of A?

Ans. $2\frac{1}{2}$ miles; A is 3 miles, and B $\frac{1}{2}$ mile from A's first position.

6. Weights of $3\frac{3}{4}$ and $4\frac{1}{4}$ oz. respectively hang by a string over a pulley, and one draws the other up; in how many seconds from the beginning of the motion would they acquire a velocity of 6 feet a second? ($g = 32$.) What circumstances, that would more or less affect the result, have you omitted in the calculation? *Ans.* 3 seconds.

7. A body whose mass is 10 lbs. is moving at the rate of 30 feet a second; what is its kinetic energy or accumulated work in foot-pounds? *Ans.* $140\frac{5}{8}$ foot-pounds.

8. A body weighing 10 lbs. slides down an inclined plane whose height is 25 feet; it reaches the foot of the plane with a velocity of 30 feet a second; how many foot-pounds of energy have been expended during the motion on friction and other resistances? *Ans.* $109\frac{3}{8}$ foot-pounds.

9. A rectangular reservoir is full of water, its length and depth being respectively 10 feet and 5 feet; a diagonal line is drawn along one of its vertical sides, dividing it into two triangles; find the magnitude of the resultant of the fluid pressures on each of these triangles (a cubic foot of water may be taken to weigh 1,000 ozs.).

Ans. The pressures are $2{,}604\frac{1}{6}$ lbs., and $5{,}208\frac{1}{3}$ lbs.

10. The pressure of air in the receiver of an air-pump is 30 inches; after a few strokes its pressure is reduced to 27 inches; what part of the original contents of the receiver has been withdrawn? *Ans.* $\frac{1}{10}$.